Institut for Matematik og Datalogi
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March 3, 2017
JFB

On-Line Algorithms – F17 – Lecture 8

Lecture, February 28

We finished chapter 6.

Lecture, March 3

We briefly covered chapter 7 and and up through the introduction to section 8.3 in chapter 8 in the textbook.

Lecture, March 8

We will finish chapter 8 and begin on the paper, "The relative worst order ratio applied to paging", by J. Boyar, L.M. Favrholdt, and K.S. Larsen, in *Journal of Computer and System Sciences*, volume 73, pages 817–843, 2007. You get this through the electronic journals SDU's library has (using the link on the course's homepage). In section 2, we will initially only consider definitions 1 and 2 and skip the others. Next we will cover up through Corollary 3 of section 4, and then section 6. Note that the slides are available through the course homepage.

Problems for March 10

1. Problems that we didn't finish from March 7.
2. Consider the Dual Bin Packing Problem from the first weekly note, and assume we are only considering fair algorithms. Consider the following adversary against a deterministic algorithm $\mathbb{A}$: Give $\mathbb{A}$ the following request sequence, divided into three phases. Phase 1 consists of n small items of size $\frac{1}{n}$. Phase 2 consists of items, one for each bin which $\mathbb{A}$ did not fill completely with size equal to the empty space in that bin,

sorted in decreasing order. After these are given, $\mathbb{A}$ has filled all bins completely and so must reject the items in Phase 3, which consists of $\frac{n^2}{4}$ items of size $\frac{1}{n}$.

To analyze this, let q denote the number of bins in $\mathbb{A}$'s configuration which have at least 2 items after the first phase.

In the case where $q < \frac{n}{4}$, we know that $\mathbb{A}$ has at least $n - q \geq \frac{3n}{4}$ bins with at most one item after Phase 1. OPT can arrange the items from Phase 1 such that half of the bins contain two items and half contain no items.

In the case where $q \geq \frac{n}{4}$, we know that $\mathbb{A}$ has at least $\frac{n}{4}$ empty bins after Phase 1. OPT places each of the items from Phase 1 in a different bin.

a. Use the adversary above to show that no fair algorithm for the dual bin packing problem is more than $\frac{8}{6+n}$-competitive (strict competitive ratio).

b. Try changing the above adversary to an adaptive on-line adversary. What result can you get?

c. Define an algorithm which uses a mixed strategy to solve the dual bin packing problem.

d. Define an algorithm which uses a behavioral strategy to solve the dual bin packing problem.

3. Do exercise 7.3 in the textbook. See page 122 for the coupon collector's problem.

   Assume that there are $k + 1$ pages in all, and obtain kH_k as a result.

4. Work out Example 8.5, and apply Yao's principle correctly to Example 8.4 in the textbook, using the distribution given there. You will not get as good a result for Example 8.4 as for Example 8.5.

5. Consider the following on-line problem: We have one processor. Jobs arrive over time; job J_j with processing time p_j arrives at time r_j. A job can be assigned to run on the processor when it arrives or any time after that.

It can also be started on the processor, stopped at some point, and restarted at some later point. No two jobs may be running at the same time. The goal is to minimize total completion time. Let C_j denote the completion time of job j. The total completion time is $\sum C_j$.

Use Yao's principle to prove a lower bound on the competitive ratio of any randomized algorithm for this problem. Consider the following probability distribution on request sequences: At time 0, a job with processing time 1 arrives. At time $\frac{1}{3}$, two jobs with processing time 0 arrive. At time 1, all of the following jobs arrive with probability p

(with probability $1 - p$ none of them arrive): 10 jobs with processing time 0 and four jobs with processing time 1.