Institut for Matematik og Datalogi
Syddansk Universitet

February 22, 2019
JFB

On-Line Algorithms – F19 – Lecture 1

Textbook

Allan Borodin and Ran El-Yaniv, *Online Computation and Competitive Analysis*, Cambridge University Press, 1998. There will be supplementary articles.

Format

The course will be taught by Joan Boyar. The lectures will be in English. The course will generally be at 14:15 on Mondays, 10:15 on Tuesdays, and 8:15 on Wednesdays, in the seminar room. The first two classes will be lectures, but after that we will alternate between lectures and discussion sections (though this will not be strict). Ignore what the official schedule about which are lectures and which are discussion sections.

There will both be assignments which you are required to turn in and other problems and exercises which you should be prepared to discuss in the discussion sections (øvelserne/træningstimerne).

The results from your required assignments will be considered along with your performance at the oral exam for your grade in the course, though the performance at the oral exam will count much more. There will be 3 assignments. The assignments must be turned in on time using the Blackboard system, submitted via the menu item "SDU Assignment". Turn in each assignment as a single PDF file. Do not use any Danish letters or other non-ASCII symbols in the name of the file. Keep the receipt it gives you proving that you turned your assignment in on time. You may work in groups of 2 to 3 students on the first and third assignments if you wish (this is encouraged). These 3 assignments are considered part of the exam and will affect your final grade, so cheating on these assignments is viewed as cheating on an exam. You are allowed to talk about course material with your fellow students, but working together on assignments with students not in your group is cheating, as is allowing other students not in your group to see your solutions. (You can, however, talk with me.) Using solutions you find elsewhere, such as on the Internet, is also cheating. You may do the assignments in either English or Danish, but if you write them by hand, please do so very neatly.

Make sure the names of everyone in your group appears on the cover page, or at the top of the first page. Turn in only one copy per group.

There will be an oral exam in June.

The weekly notes and other information about the course is available through Blackboard or the URL:

http://www.imada.sdu.dk/~joan/online/

Please do not hesitate to contact me if you have questions concerning the course. I have office hours on Mondays and Thursdays from 9:00 to 9:45.

Lecture, February 4

We will begin with an introduction to the course. Then, we will begin on chapter 1 in the textbook, probably covering up through Theorem 1.1.

Lecture, February 5

We will continue with chapter 1 in the textbook.

Problems for February 6

Note that we may not finish all of these problems. If not, we will continue on February 12.

1. Do Exercise 1.2 in the textbook (assume the static problem).

2. For the proof of Theorem 1.1, work out the details for insertion of an item.

3. This is a modification of Exercise 1.4. The algorithm *fractional MTF* uses a parameter $d \geq 1$. Let MTF_d be the variant of MTF that behaves as follows. When an item is accessed or inserted at position i, it moves it $\frac{i}{d}-1$ positions closer to the front of the list. Show that $\mathrm{MTF}_d(\sigma) \leq d(2\cdot$

$\mathrm{OPT}_C + \mathrm{OPT}_A - n$), where OPT_A is the total number of transpositions performed by OPT.

Hint: Use d times the number of inversions for your potential function.

4. Do Exercise 1.6 in the textbook.

5. Consider the algorithm MTF2 for list accessing, which moves an item to the front of the list every second time it accesses it doing the move on every odd numbered access. Let the list consist of the items $\{a_1, a_2, ..., a_n\}$. Show that on the following sequence (repeated s times), the ratio of MTF2's performance to OPT's is at least $\frac{5}{2}$.

$$\langle (a_1, a_2, \ldots, a_n, a_1^3, a_2^3, \ldots, a_n^3, a_n, a_{n-1}, \ldots, a_1, a_n^3, a_{n-1}^3, \ldots, a_1^3)^s \rangle$$

   Note that this observation reveals an error in Exercise 1.5 in the textbook.

   Can you show something similar if MTF2 moves on every even access, instead of every odd numbered access?

6. Show that MTF2 is 3-competitive using the potential function method. This can be done counting inversions, as for MTF, but giving the inversions weights based on how many requests to the item are necessary before MTF2 moves it to the front (the weights are 1 or 2). Then the two cases of whether or not MTF2 moves an item to the front are handled separately in the amortized analysis.

7. For the on-line Dual Bin Packing Problem, there are n bins of unit size. The request sequence contains items of sizes $0 \leq s \leq 1$ which are to be placed in the bins. The goal is to accept and pack as many items in the request sequence as possible subject to the constraint that no bin should contain items whose sizes add up to more than 1. This is a maximization problem.

   (a) Give a definition of the competitive ratio for this problem which gives ratios of at most 1.

   (b) Consider the algorithm First-Fit, which places an item in the first bin in which it fits. First-Fit is a *fair* algorithm, an algorithm which never rejects an item if it fits in one of the bins when it is rejected. Define two more fair algorithms and one which is not fair.

(c) Assume that $\frac{1}{k}$ is the size of the smallest item in any sequence. Prove an upper bound of $\frac{2-\frac{1}{k}}{k}$ on First-Fit's competitive ratio on sequences of this type (assume that OPT must also be fair). Is First-Fit competitive? Prove a lower bound of $\frac{1}{k}$ on First-Fit's competitive ratio on sequences of this type.