

Exercises Nov. 18 and 21

1. ADAPTED FROM JUNE 2006, 3

Convert the following predicate logic expression to clausal form:

$$\forall X(\forall Y(p(X, Y) \Leftrightarrow \neg(\exists Z(q(X, Z))))))$$

Document the steps of your conversion.

2. ADAPTED FROM JANUARY 2000, 5A

Rewrite the following logical expression to clausal form:

$$\neg(\forall X)(p(X) \Rightarrow ((\forall Y)(p(Y) \Rightarrow p(f(X, Y))) \wedge \neg(\forall Z)(q(X, Z) \Rightarrow p(Z))))$$

3. ADAPTED FROM JUNE 2001, 2A

Rewrite the following logical expression to clausal form:

$$\exists X(p(X) \Rightarrow \forall Y(p(Y) \Rightarrow p(f(X, Y)))) \wedge (\forall Z(q(X, Z) \Rightarrow p(Z)))$$

4. ADAPTED FROM JANUARY 2001, 2A

Rewrite the following logical expression to clausal form:

$$\forall X \forall Y (s(X, Y) \Rightarrow (\neg(m(X) \vee \forall Z(t(X, Z) \wedge (\neg m(Z))))))$$

5. ADAPTED FROM JUNE 2005, 2

In this question, we consider sequences of elements from a set of size three. For concreteness, let the set be $S = \{1, 2, 3\}$, and the sequences be strings over S . In such a string, two identical nonempty neighboring substrings are said to form a repetition. As an example, the following string contains the two underlined repetitions:

311321231232.

A string having no repetitions is said to be repetition-free. The task of this exercise is to develop a PROLOG predicate which generates all repetition-free strings over S of a given length. Strings will be represented as lists of integers from S .

- Implement a PROLOG predicate `frontRep(L)` which is true if and only if there is a repetition starting at the front of the list L . *Hint: standard predicates (from textbook or standard library) on lists may come in handy.*
- Implement a PROLOG predicate `repFree(X, N)` which is true if and only if X is a repetition-free list of elements in S and has length N . The predicate must be able to generate (as instantiations of X) all repetition-free lists of length N , by repeated use of `;`.
- Implement a PROLOG predicate `countLessThanEq(N, R)` which is true if and only if R is the number of repetition-free lists of elements in S of length less than or equal to N . The number of repetition-free lists of length zero is defined to be one.