

## DM205 – On-Line Algorithms – Lecture 1

### Lecture, August 30

- Introduction to the course.
- Borodin & El-Yaniv, Sections 1.0–1.3.

### Lecture, August 31

- Borodin & El-Yaniv, Sections 1.4–1.6.

### Exercises, September 2

If we do not finish all of these problems, we will continue next time. All references are to the textbook by Borodin & El-Yaniv unless otherwise stated.

1. Exercise 1.2 (assume the static problem).
2. For the proof of Theorem 1.1, work out the details for insertion of an item.
3. This is a modification of Exercise 1.4. The algorithm *fractional MTF* uses a parameter  $d \geq 1$ . Let  $\text{MTF}_d$  be the variant of MTF that behaves as follows. When an item is accessed or inserted at position  $i$ , it moves it  $\frac{i}{d} - 1$  positions closer to the front of the list. Show that  $\text{MTF}_d(\sigma) \leq d(2 \cdot \text{OPT}_C + \text{OPT}_A - n)$ , where  $\text{OPT}_A$  is the total number of transpositions performed by OPT.

Hint: Use  $d$  times the number of inversions for your potential function.

4. Exercise 1.6.
5. Consider the algorithm MTF2 for list accessing, which moves an item to the front of the list every second time it accesses it. Let the list consist of the items  $\{a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n\}$ . Show that on the following sequence (repeated  $s$  times), the ratio of MTF2's performance to OPT's is at least  $\frac{7}{3}$ .

$$\begin{aligned} &\langle a_1, a_2, \dots, a_n, \quad b_n, b_n, b_{n-1}, b_{n-1}, \dots, b_1, b_1, \\ &\quad a_n, a_{n-1}, \dots, a_1, b_n, b_n, b_{n-1}, b_{n-1}, \dots, b_1, b_1, \\ &\quad b_1, b_2, \dots, b_n, \quad a_n, a_n, a_{n-1}, a_{n-1}, \dots, a_1, a_1, \\ &\quad b_n, b_{n-1}, \dots, b_1, a_n, a_n, a_{n-1}, a_{n-1}, \dots, a_1, a_1 \rangle \end{aligned}$$

Hint: Assume that MTF2 starts with the list ordered as

$$\langle a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n \rangle$$

while the algorithm it is competing against starts with

$$\langle a_1, a_2, \dots, a_n, b_n, b_{n-1}, \dots, b_1 \rangle$$

Note that this observation is due to Ansgar Grüne and his students and reveals an error in Exercise 1.5 in the textbook.

6. For the on-line Dual Bin Packing Problem, there are  $n$  bins of unit size. The request sequence contains items of sizes  $0 \leq s \leq 1$  which are to be placed in the bins. The goal is to accept and pack as many items in the request sequence as possible subject to the constraint that no bin should contain items whose sizes add up to more than 1. This is a maximization problem.
- (a) Give a definition of the competitive ratio for this problem which gives ratios of at most 1.
  - (b) Consider the algorithm First-Fit, which places an item in the first bin in which it fits. First-Fit is a *fair* algorithm, an algorithm which never rejects an item if it fits in one of the bins when it is rejected. Define two more fair algorithms and one which is not fair.
  - (c) Assume that  $\frac{1}{k}$  is the size of the smallest item in any sequence. Prove an upper bound of  $\frac{2-\frac{1}{k}}{k}$  on First-Fit's competitive ratio on sequences of this type (assume that OPT must also be fair). Is First-Fit competitive? Prove a lower bound of  $\frac{1}{k}$  on First-Fit's competitive ratio on sequences of this type.