Department of Mathematics and Computer Science
University of Southern Denmark

September 7, 2010
JFB, KSL

DM205 – On-Line Algorithms – Lecture 4

Lecture, September 6

- Borodin & El-Yaniv, Sections 2.2–2.5.

Lecture, September 9

- Borodin & El-Yaniv, Sections 12.0–12.2.

Lecture, September 14

- Borodin & El-Yaniv, Chapter 3.

Exercises, September 16

All references are to the textbook by Borodin & El-Yaniv unless otherwise stated.

1. (Easy) Show that the makespan problem for identical machines is NP-hard.
2. Suppose that GREEDY is allowed n identical machines, while OPT is only allowed to use $m < n$ machines. Give a sequence showing that the ratio of GREEDY's performance to OPT's can be at least $1 + \frac{m-1}{n}$ for the makespan problem. Then show that GREEDY can always achieve this ratio against such a bounded OPT.
3. Consider remark 12.1 on page 208. What is meant here? Why is there no problem if the loads can be greater than 1? (Do not try to prove the desired result for loads of at most 1.)
4. Define POST-GREEDY with release dates as the algorithm which assigns a new job (given at its release date) to the first processor which becomes free. (Jobs have processing times which may be unknown, and only one job may be running on a processor at a time. There are m processors.) Show that POST-GREEDY is $(2 - \frac{1}{m})$-competitive.

5. Consider the Dual Bin Packing Problem from the first weekly note, and assume we are only considering fair algorithms. Consider the following adversary against a deterministic algorithm $\mathbb{A}$: Give $\mathbb{A}$ the following request sequence, divided into three phases. Phase 1 consists of n small items of size $\frac{1}{n}$. Phase 2 consists of items, one for each bin which $\mathbb{A}$ did not fill completely with size equal to the empty space in that bin, sorted in decreasing order. After these are given, $\mathbb{A}$ has filled all bins completely and so must reject the items in Phase 3, which consists of $\frac{n^2}{4}$ items of size $\frac{1}{n}$.

   To analyze this, let q denote the number of bins in $\mathbb{A}$'s configuration which have at least 2 items after the first phase.

   In the case where $q < \frac{n}{4}$, we know that $\mathbb{A}$ has at least $n - q \geq \frac{3n}{4}$ bins with at most one item after Phase 1. OPT can arrange the items from Phase 1 such that half of the bins contain two items and half contain no items.

   In the case where $q \geq \frac{n}{4}$, we know that $\mathbb{A}$ has at least $\frac{n}{4}$ empty bins after Phase 1. OPT places each of the items from Phase 1 in a different bin.

   Use the adversary above to show that no fair algorithm for the dual bin packing problem is more than $\frac{8}{6+n}$-competitive (strict competitive ratio).