

DM205 – On-Line Algorithms – Lecture 6

Lecture, September 14

- Borodin & El-Yaniv, Chapter 3 and Chapter 4 through Theorem 4.1.

Lecture, September 20

- Borodin & El-Yaniv, Chapter 4.

Lecture, September 23

- Starting on “The Relative Worst Order Ratio Applied to Paging” (available via the course home page).

Exercises, September 27

All references are to the textbook by Borodin & El-Yaniv unless otherwise stated.

1. Exercise 4.2.
2. Exercise 4.5.
3. Exercise 4.6.
4. Let OPT_k denote OPT using a cache of size k . Consider the function

$$f(\sigma) = \min\{k \mid \forall k' > k: \text{OPT}_{k'}(\sigma) = \text{OPT}_k(\sigma)\}$$

That is, the smallest cache size for which it does not help OPT to get a larger cache. Try to define f without any reference to OPT, i.e., by only considering properties of σ .

5. Consider an algorithm with look-ahead s , meaning that when deciding what to do about the current page request, the algorithm can see the next s requests before deciding what to do.
 - (a) Prove that any such deterministic algorithm has competitive ratio at least k .

- (b) Consider $\text{LRU}(s)$, the algorithm which uses the LRU rule, ignoring (that is, never evicting) any page in the next s requests. Show that it does at least as well as LRU on any request sequence (assuming they start with the same pages in fast memory).