DM205 – On-Line Algorithms – Lecture 7

Lecture, September 20

- Borodin & El-Yaniv, Chapter 4.

Lecture, September 23

- Starting on "The Relative Worst Order Ratio Applied to Paging" (available via the course home page).

  In section 2, we will initially only consider Definitions 1 and 2 and skip the others. Next, we will cover up through Corollary 3 of Section 4, and then Section 6.

Lecture, September 28

- Finishing "The Relative Worst Order Ratio Applied to Paging".

  We will cover Sections 3 and 6, the definitions for relatedness and weakly comparable in Section 2, and Theorem 7 of Section 5.

Exercises, September 30

All references are to the textbook by Borodin & El-Yaniv unless otherwise stated.

1. Show that with the relative worst order ratio, for a given problem, the ordering as to which algorithms are better than which is transitive: Show that if $\mathrm{WR}_{\mathbb{A},\mathbb{B}} \geq 1$ and $\mathrm{WR}_{\mathbb{B},\mathbb{C}} \geq 1$, then $\mathrm{WR}_{\mathbb{A},\mathbb{C}} \geq \mathrm{WR}_{\mathbb{B},\mathbb{C}}$. Furthermore, show that if $\mathrm{WR}_{\mathbb{A},\mathbb{B}}$ is bounded from below by some constant, then $\mathrm{WR}_{\mathbb{A},\mathbb{C}} \geq \mathrm{WR}_{\mathbb{A},\mathbb{B}}$.

2. Lemma 4 in the article "The relative worst order ratio applied to paging" does not hold if the conservative algorithm is allowed look-ahead. How do you know this? Where does the proof fail?

3. Find another sequence which would separate LRU's and FWF's behavior under the relative worst order ratio. (It is not necessary to get as large a ratio as the one in the article. Try for $\frac{3}{2}$.)

4. Try defining an algorithm which is based on FIFO and uses look-ahead. What is its relative worst order ratio compared to FIFO? To LRU?

5. Consider the algorithm for dual bin packing (fixed number of bins, maximizing the number of accepted items) that behaves exactly as First-Fit would unless the item x is larger than $\frac{1}{2}$ and would be placed in the last bin, bin n. The algorithm FF_n rejects such an item and is thus not fair.

   Show that FF_n is better than First-Fit, according to the relative worst order ratio.

6. Compare MTF and TRANS for the list accessing problem, using the relative worst order ratio.