Department of Mathematics and Computer Science
University of Southern Denmark

September 28, 2010
JFB, KSL

DM205 – On-Line Algorithms – Lecture 9

Lecture, September 28

- Finishing "The Relative Worst Order Ratio Applied to Paging".

  We covered Section 3, the definitions for relatedness and weakly comparable in Section 2, and Theorem 7 of Section 5.

Lecture, October 4

- Borodin & El-Yaniv, Chapter 6 and 7, skipping most proofs.

Lecture, October 7

- Borodin & El-Yaniv, Chapter 7 and 8, skipping most proofs.

Exercises, October 11

All references are to the textbook by Borodin & El-Yaniv unless otherwise stated.

1. Exercise 6.1.
2. (Part of Exercise 6.4.) Show that the algorithm PERM_π is neither a marking algorithm nor a conservative algorithm. Try using $N = k+2$.
3. In the absent minded driver problem, is $\frac{1}{2}$ the optimal value for the behavioral strategy?
4. Exercise 6.5. First find an example where there are fewer faults. Then fix this, either by prolonging or simply rearranging.
5. Exercise 6.6.
6. Consider the Dual Bin Packing Problem from the first weekly note, and assume we are only considering fair algorithms. Recall that for this problem there is a fixed collection of n bins, the objective is to pack as many items as possible, and fairness means that one cannot

reject an item which could be fit into some bin at the time of the request.

Consider the following adversary against a deterministic algorithm $\mathbb{A}$: Give $\mathbb{A}$ the following request sequence, divided into three phases. Phase 1 consists of n small items of size $\frac{1}{n}$. Phase 2 consists of items, one for each bin which $\mathbb{A}$ did not fill completely with size equal to the empty space in that bin, sorted in decreasing order. After these are given, $\mathbb{A}$ has filled all bins completely and so must reject the items in Phase 3, which consists of $\frac{n^2}{4}$ items of size $\frac{1}{n}$.

To analyze this, let q denote the number of bins in $\mathbb{A}$'s configuration which have at least 2 items after the first phase.

In the case where $q < \frac{n}{4}$, we know that $\mathbb{A}$ has at least $n - q \geq \frac{3n}{4}$ bins with at most one item after Phase 1. Here, OPT can arrange the items from Phase 1 such that half of the bins contain two items and half contain no items.

In the case where $q \geq \frac{n}{4}$, we know that $\mathbb{A}$ has at least $\frac{n}{4}$ empty bins after Phase 1. Here, can OPT place each of the items from Phase 1 in a different bin.

a) Use the adversary above to show that no fair algorithm for the dual bin packing problem is more than $\frac{8}{6+n}$-competitive (strict competitive ratio).

b) Assume that the online algorithm uses randomization. What results can you establish against ADOF, ADON, and OBL based on the adversarial sequence above?

c) Suggest an algorithm which uses a mixed strategy to solve the dual bin packing problem.

d) Suggest an algorithm which uses a behavioral strategy to solve the dual bin packing problem.