# Optimizing the Search for Repairs from Active Integrity Constraints

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LabMAg Seminar September 18th, 2013

Active integrity constraints

- Active integrity constraints
- 2 Parallelization

- Active integrity constraints
- Parallelization
- Stratification

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- 4 Conclusions

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# Example: family relations

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### Definition (Flesca2004)

An Active integrity constraint is a formula of the form

$$L_1,\ldots,L_m\supset\alpha_1\mid\ldots\mid\alpha_k$$

where  $\{\alpha_1^D, \ldots, \alpha_k^D\} \subseteq \{L_1, \ldots, L_m\}$ .

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$$siblingOf(x, y) \land \neg siblingOf(y, x) \supset + siblingOf(y, x)$$

#### An invalid AIC

 $\mathsf{siblingOf}(x,y) \land \neg \mathsf{siblingOf}(y,x) \supset -\mathsf{siblingOf}(x,y) \mid +\mathsf{Parent}(x)$ 

$$L_1,\ldots,L_m\supset\alpha_1\mid\ldots\mid\alpha_k$$

## A generic AIC

$$L_1, \ldots, L_m \supset \alpha_1 \mid \ldots \mid \alpha_k$$

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- $\{\alpha_1^D, \dots, \alpha_k^D\}$  are updatable literals

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#### Definition

A repair is a weak repair that is minimal w.r.t. inclusion.

#### Inconsistency

siblingOf(John, Mary)

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#### Another repair

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#### A weak repair

+siblingOf(Mary, John), +Parent(John)

#### **Inconsistency**

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#### Not a weak repair

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# Finding repairs

### "Algorithm"

- **1** Choose a set  $\mathcal{U}$  of update actions (based on  $\mathcal{I}$ )
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- $\odot$  Check if all AICs in  $\eta$  hold

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Finding weak repairs is NP-complete.

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- founded repairs take into account the actions in the head
- justified repairs avoid justification circles

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A set of update actions  $\mathcal U$  is founded w.r.t.  $\mathcal I$  and  $\eta$  if, for every  $\alpha \in \mathcal U$ , there is a rule  $r \in \eta$  such that  $\alpha \in \operatorname{head}(r)$  and

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#### Definition

A (weak) repair that is founded is called a founded (weak) repair.

### Example

Take 
$$\mathcal{I} = \{a, b\}$$
 and

$$r_1: a, \text{ not } b \supset -a$$

$$r_2$$
:not  $a, b \supset -b$ 

$$r_3: a, \text{ not } c \supset +c$$

$$r_4:b$$
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Again  $\{-a, -b\}$  is a founded repair that is not justified.

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Deciding whether there is a justified (weak) repair for  $\mathcal I$  and  $\eta$  is  $\Sigma^2_P$ -complete.

Stratification

### Outline

- Active integrity constraints
- Parallelization

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- Not affected by the database.

### Example

Consider the following AICs:

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Then  $\{r_1, r_2\} \perp \!\!\! \perp \{r_3\}$ .

#### Theorem

Let  $\eta = \eta_1 \cup \eta_2$  with  $\eta_1 \perp \!\!\! \perp \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}$  be a weak repair for  $\mathcal{I}$  and  $\eta$ .

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Furthermore, if every action in  $\mathcal{U}$  affects a literal in the body of a rule in  $\eta$ , then  $\mathcal{U} = \mathcal{U}_1 \cup \mathcal{U}_2$ . This hypothesis is (very) reasonable in practice.

### Proof (sketch).

Given  $r_1 \in \eta_1$  and  $r_2 \in \eta_2$ , changing the logical values of literals in the body of  $r_1$  cannot affect the semantics of  $r_2$  and vice-versa.

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For foundedness, take  $\alpha \in \mathcal{U}_1$ . Since  $\mathcal{U}$  is founded, there is a rule  $r \in \eta$  such that  $\mathcal{I} \circ (\mathcal{U} \setminus \{\alpha\}) \not\models r$ . Necessarily  $r \in \eta_1$  and  $\mathcal{I} \circ (\mathcal{U}_1 \setminus \{\alpha\}) \not\models r$ .

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This means that we can parallelize the search for repairs without losing solutions.

### Example

Let  $\mathcal{I} = \{a, b, c, d\}$  and consider the following set of AICs  $\eta$ :

$$r_1: a, \text{ not } b \supset -a$$
  $r_2: b, c \supset -c$   $r_3: d \supset -d$ 

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Since  $\eta_1 = \{r_1, r_2\}$  and  $\eta_2 = \{r_3\}$  are independent, the theorem guarantees that  $\mathcal{U}_1 = \{-c\}$  and  $\mathcal{U}_2 = \{-d\}$  are founded repairs for  $\mathcal{I}$  and  $\eta_1$  or  $\eta_2$ , respectively.

#### Theorem

Let  $\eta = \eta_1 \cup \eta_2$  with  $\eta_1 \perp \!\!\! \perp \eta_2$ ;  $\mathcal I$  be a database; and  $\mathcal U_i$  be weak repairs for  $\mathcal I$  and  $\eta_i$ , for i=1,2, such that all actions in  $\mathcal U_i$  affect a literal in the body of a rule in  $\eta_i$ .

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#### Then:

- $\mathcal{U}$  is a weak repair for  $\mathcal{I}$  and  $\eta$ ;
- if each  $U_i$  is a repair, then so is  $U_i$ ;
- if each  $U_i$  is founded, then so is U;
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This means that parallelization of the search does not add "new" (false) solutions.

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In order to split a set  $\eta$  into independent sets, consider the relation  $\Delta$  of dependence. This relation is reflexive and symmetric. Therefore its transitive closure  $\Delta$ <sup>+</sup> is an equivalence relation. The

quotient set  $^{\eta}/_{\text{AL}^+}$  is the finest partition of  $\eta$  in independent sets, and can be computed efficiently.

### Outline

- Active integrity constraints
- Stratification

#### Definition

AIC  $r_1$  precedes AIC  $r_2$ ,  $r_1 \prec r_2$ , if some action in the head of  $r_1$  affects a literal in the body of  $r_2$ .

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- $\langle ^{\eta}/_{\approx}, \preceq \rangle$  is a partial order, where  $\preceq$  is the transitive closure of  $\prec$  and  $\approx$  is the induced equivalence relation.

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(Similar to stratified negation in logic programming...)

#### Example

Consider the following set of AICs  $\eta$ .

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$$r_3$$
: not  $a, c, d \supset -c \mid -d$ 

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The precedence relation is summarized in the following diagram.

Stratification

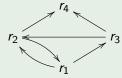
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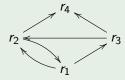


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The precedence relation is summarized in the following diagram.



The equivalence classes are  $\eta_1 = \{r_1, r_2, r_3\}$  and  $\eta_2 = \{r_4\}$ .

# Precedence vs. stratification (I)

#### Theorem

Active integrity constraints

Let  $\eta_1, \eta_2 \in {}^{\eta}/_{\approx}$  with  $\eta_1 \prec \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}$  be a weak repair for  $\mathcal{I}$  and  $\eta_1 \cup \eta_2$ .

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However, some results do not hold:

- ullet it may happen that  ${\cal U}$  is a repair, but  ${\cal U}_1$  and/or  ${\cal U}_2$  are not;
- there may be a weak (founded, justified) repair  $\mathcal{U}_1$  for  $\mathcal{I}$  and  $\eta_1$  that is not a subset of any weak repair for  $\mathcal{I}$  and  $\eta_1 \cup \eta_2$ .

#### Example

Let  $\mathcal{I} = \emptyset$  and consider the following active integrity constraints.

$$r_1$$
: not  $a \supset +a$   $r_2$ : not  $b, c \supset +b$   $r_3$ :  $b$ , not  $c \supset +c$   $r_4$ :  $a$ , not  $b$ , not  $c$ ,  $d \supset -d$   $r_5$ :  $a$ , not  $b$ , not  $c$ , not  $d \supset +d$ 

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Taking 
$$\eta_1 = \{r_1, r_2, r_3\}$$
 and  $\eta_2 = \{r_4, r_5\}$ , one has  $\eta_1 \prec \eta_2$ .

Stratification

### Precedence vs. stratification (III)

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Taking  $\eta_1 = \{r_1, r_2, r_3\}$  and  $\eta_2 = \{r_4, r_5\}$ , one has  $\eta_1 < \eta_2$ . Furthermore,  $\{+a\}$  and  $\{+a,+b,+c\}$  are weak repairs for  $\mathcal{I}$  and  $\eta_1$ , the first of which is a repair. However, the only repair for  $\mathcal{I}$ and  $\eta_1 \cup \eta_2$  is  $\{+a, +b, +c\}$ .

#### Theorem

Let  $\eta_1$ ,  $\eta_2$  and  $\mathcal{I}$  be as before;  $\mathcal{U}_1$  be a weak repair for  $\mathcal{I}$  and  $\eta_1$ ;  $\mathcal{U}_2$  be a weak repair for  $\mathcal{I} \circ \mathcal{U}_1$  and  $\eta_2$ ; such that every action in  $\mathcal{U}_i$  occurs in the head of a rule in  $\eta_i$ .

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- Active integrity constraints
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- 4 Conclusions

Conclusions

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- (More) practical evaluation
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- Generalizations of AICs outside the database world

# Thank you.