Department of Mathematics and Computer Science
University of Southern Denmark, Odense

March 18, 2005
LMF

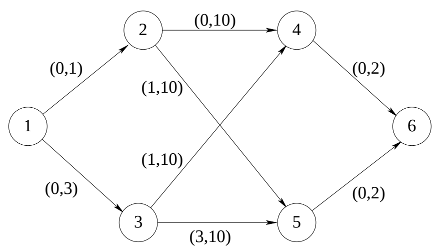
DM69 — Lecture 8

Lecture 8 — March 30

- Matchings in general graphs: Sections 10.4–10.5 in Papadimitriou and Steiglitz.
- The primal-dual algorithm applied to the assignment problem and the transportation problem: Section 3.12 in Bang-Jensen and Gutin.

Problems for April 1

1. Problems 3.49 and 3.50 in Bang-Jensen and Gutin.
2. Problem 3.52 in Bang-Jensen and Gutin.
   If you cannot find a graph without parallel arcs, try to find a multigraph.
3. Equivalence of minimum cost flow algorithms.
   (a) Consider the minimum cost flow problem below with $b(1) = -b(6) = 4$ and $b(2) = b(3) = b(4) = b(5) = 0$. The numbers next to the arcs are (c, u).

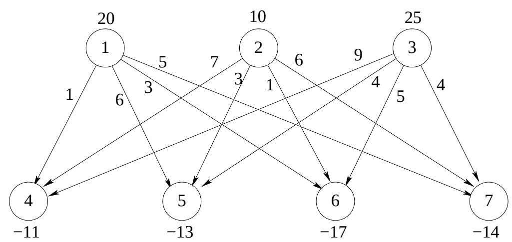


   Apply the Buildup Algorithm to this network. Show that it performs four augmentations from vertex 1 to vertex 6.

   (b) Add the arc $(1, 6)$ with sufficiently large cost and $u_{16} = 4$. Setting $x_{16} = 4$ and $x_{ij} = 0$ for all other arcs ij in the network gives a feasible flow. With this flow as the initial flow, apply the Cycle Canceling Algorithm and always augment flow along a negative cycle with minimum cost. Show that this algorithm also performs four unit flow augmentations from vertex 1 to vertex 6 along the same paths as in part (a) and in the same order, except that the flow returns to vertex 1 through the arc $(6, 1)$ in the residual network.

4. More for less.

(a) Consider the minimum cost flow problem shown below. The numbers next to arcs are costs, and the numbers next to vertices are balances. All capacities are infinite.



Show that the following flow is an optimal flow. $x_{14} = 11$, $x_{16} = 9$, $x_{25} = 2$, $x_{26} = 8$, $x_{35} = 11$, $x_{37} = 14$, and $x_{ij} = 0$ for all other arcs ij. What is the cost of the flow?

(b) Suppose that we increase $b(2)$ by 2 units, decrease $b(4)$ by 2 units, and reoptimize the flow. Show that the cost of the flow decreases.

Have a nice Easter holiday!