

## DM833 – Weekly Note 2

### Lectures in week 16

#### Monday, April 15

- Although the vertex cover problem is NP-hard for general graphs, there are graph classes that allow for efficient algorithms.  
Design an algorithm that finds an optimal vertex cover for a tree in linear time.
- Exercise 2.1
- Exercise 2.2. Is the lower bound of  $1/2$  tight?
- Exercise 2.8
- Section 3.1: The Steiner Tree Problem

#### Tuesday, April 16

- Make an outline of a 10 minutes presentation of the vertex cover and set cover problems.  
This is meant as a preparation for the exam, so you should choose the part of the material that best communicates what you know about the topic.  
Check the course homepage for advice on presenting exam questions.
- Exercise 2.6
- Section 4.0 and 4.1: Multiway Cut

#### Wednesday, April 17

- Consider the following algorithm for finding a TSP tour in a graph with metric edge weights:  
Vertices are added to the cycle one by one.  
In each step, the vertex added is a vertex  $v$  whose distance to any of the vertices already in the cycle is minimum.  
Assume that the vertex closest to  $v$  is  $u$ . Then,  $v$  is added to the cycle just after  $u$ .  
Prove that the algorithm is a 2-approximation algorithm.  
Hint: Note the similarity to Prim's algorithm for finding a minimum spanning tree.
- Let  $G$  be a complete undirected graph with nonnegative edge weights. Consider the following transformation:  
Let  $W$  be the maximum weight in  $G$ .  
For each edge  $e$ , add  $W$  to the weight of  $e$ .  
Call the resulting weighted graph  $G'$ .
  - Argue that the weights in  $G'$  are metric.
  - Argue that a TSP tour in  $G$  is optimal, iff the corresponding tour in  $G'$  is optimal for  $G'$ .
  - Does this contradict Theorem 3.6?
- Section 5.0 and 5.1: The  $k$ -Center Problem and parametric pruning