DM833 – Week 17

Exercises for Tuesday, April 22

1. Consider the following algorithm for finding a TSP tour in a graph with metric edge weights:

   Vertices are added to the cycle one by one.
   In each step, the vertex added is a vertex v whose distance to any of the vertices already in the cycle is minimum.
   Assume that the vertex closest to v is u. Then, v is added to the cycle just after u.

   Prove that the algorithm is a 2-approximation algorithm.
   Hint: Note the similarity to Prim's algorithm for finding a minimum spanning tree.

2. Let G be a complete undirected graph with nonnegative edge weights. Consider the following transformation:

   Let W be the maximum weight in G.
   For each edge e, add W to the weight of e.
   Call the resulting weighted graph G'.

   Argue that the weights in G' are metric.

   Argue that a TSP tour in G is optimal, iff the corresponding tour in G' is optimal for G'.

   Does this contradict Theorem 3.6?

3. Describe an algorithm for finding an Euler tour in a graph where all vertices have even degree.