Department of Mathematics and Computer Science
University of Southern Denmark, Odense

April 11, 2013
LMF

DM833 – Week 15

Monday, April 7

Lecture Subsections 1.0-1.1: Introduction to approximation algorithms, with Vertex Cover as an example

Tuesday, April 8

Lecture Subsections 2.0-2.1: Set Cover and the Greedy Algorithm

Exercises Exercise 1.1

Friday, April 11

Lecture Subsection 3.2: TSP

Exercises

1. Exercise 1.3 (In the 2001 printing of the book, there is a typo in the hint: $|S|$ should be replaced by $\lceil |S|/2 \rceil$.)
2. Assume that you have an algorithm for finding a minimum vertex cover in a graph. Explain how you can use the algorithm for finding a maximum independent set.

   Does this mean that you can use Algorithm 1.2 for approximating a maximum independent set? (Hint: what approximation factor could you obtain?)
3. Although the vertex cover problem is NP-hard for general graphs, there are graph classes that allow for efficient algorithms.

   Design an algorithm that finds an optimal vertex cover for a tree in linear time.

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DM833 – Week 16

Monday, April 14

Lecture Section 3.1: The Steiner Tree problem

Exercises

1. Exercise 2.1
2. Exercise 2.2. Is the lower bound of $1/2$ tight?

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DM833 – Week 17

Tuesday, April 22

Lecture Sections 4.0–4.1: Multiway cut

Exercises

1. Exercise 2.8
2. Consider the following algorithm for finding a TSP tour in a graph with metric edge weights:

   Vertices are added to the cycle one by one.
   In each step, the vertex added is a vertex v whose distance to any of the vertices already in the cycle is minimum.
   Assume that the vertex closest to v is u. Then, v is added to the cycle just after u.

   Prove that the algorithm is a 2-approximation algorithm.
   Hint: Note the similarity to Prim's algorithm for finding a minimum spanning tree.
3. Let G be a complete undirected graph with nonnegative edge weights. Consider the following transformation:

   Let W be the maximum weight in G.
   For each edge e, add W to the weight of e.
   Call the resulting weighted graph G'.

   Argue that the weights in G' are metric.

   Argue that a TSP tour in G is optimal, iff the corresponding tour in G' is optimal for G'.

   Does this contradict Theorem 3.6?

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May 4, 2013
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DM833 – Week 18

Monday, April 28

Lecture Sections 5.0-5.1: The k-Center problem and parametric pruning

Exercises

1. Let G be a complete undirected graph with nonnegative edge weights. Consider the following transformation:

   Let W be the maximum weight in G.
   For each edge e, add W to the weight of e.
   Call the resulting weighted graph G'.

   On Tuesday, April 22, we proved that the weights in G' are metric.

   - Argue that a TSP tour in G is optimal, iff the corresponding tour in G' is optimal for G'.
   - Does this contradict Theorem 3.6?
   - What about using the metric closure of G instead of G' (as we did for the Steiner tree problem)?

2. Describe an algorithm for finding an Euler tour in a graph where all vertices have even degree.

3. Exercise 3.3

Wednesday, April 30

Lecture

- Theorem 5.7
- Section 8.1: Knapsack — a pseudo-polynomial dynamic programming algorithm

Exercises Exercise 4.2

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May 10, 2013
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DM833 – Week 19

Friday, May 9

Lecture

- Section 8.2: A FPTAS for Knapsack
- Section 8.3: Strong NP-hardness
- Section 9.0: Introduction to Bin Packing

Exercises

1. Describe an efficient implementation of Algorithm 5.3. Hint: Is it necessary to construct the square of G_j explicitly?
2. Exercise 5.1

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May 14, 2013
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DM833 – Week 20

Monday, May 12

Lecture

- Section 9.1: An asymptotic PTAS for bin packing

Exercises

1. Exercise 8.1
2. Exercise 8.2

Wednesday, May 14

Lecture

- Sections 12.1 and 12.3
- Section 13.1 up to Lemma 13.2

Exercises

1. Give an optimal Knapsack algorithm with running time $O(nB)$ using dynamic programming.
2. Exercise 8.4
3. Explain the proof of Theorem 8.5
4. Explain the proof of Corollary 8.6

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DM833 – Week 21

Tuesday, May 20

Lecture

- A short recap of Section 13.1 up to Lemma 13.2
- Lemma 13.2 and Theorem 13.3

Exercises

1. Exercise 9.1
   Hint: It is sufficient to use three different item sizes.
   If you cannot find a sequence giving a ratio of $\frac{5}{3}$, try to find a sequence with just two item sizes giving a ratio of $\frac{3}{2}$.
2. Exercise 9.2
3. Exercise 9.4
4. Exercise 9.5

Wednesday, May 21

Lecture

- Section 13:2: Dual Fitting applied to Constrained Set Multicover

Exercises

1. Exercise 13.1
2. Exercise 13.2
3. Exercise 13.3

Friday, May 23

Lecture

- Chapter 14: LP-Rounding Applied to Set Cover

Exercises

1. Exercise 13.4.1

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May 28, 2013
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DM833 – Week 22

Monday, May 26

Lecture

- Chapter 15: The Primal-Dual schema applied to Set Cover

Exercises

1. Example 14.3 uses an instance with n^k elements. Could the instance be simplified to use fewer elements and still and still give the same factor? It should be possible to get down to n elements.
2. Exercise 14.1

Wednesday, May 28

Exercises

1. Write an LP-formulation of the vertex cover problem (unweighted version). Write the dual problem as well. What combinatorial problem does the dual problem correspond to?
2. Exercise 14.3. Only the part about Set MultiCover.
3. Exercise 14.4.
4. Exercise 14.5
5. Exercise 15.5.
   Note that what you are asked to do in the first part of the exercise is to find a primal-dual algorithm with an approximation guarantee of $\frac{1}{2}$.
   Hint 1: Since this is a maximization problem, the primal and dual problems swap roles compared to what we did for the set cover problem.
   Hint 2: When choosing an unsatisfied constraint, choose one with maximum right-hand side.