DM811

Heuristics for Combinatorial Optimization

# Examples

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# Outline

# Examples

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# Single Machine Total Weighted Tardiness

**Given:** a set of $n$ jobs $\{J_1, \dots, J_n\}$ to be processed on a single machine and for each job $J_i$ a processing time $p_i$, a weight $w_i$ and a due date $d_i$.

**Task:** Find a schedule that minimizes
the total weighted tardiness $\sum_{i=1}^{n} w_i \cdot T_i$
where $T_i = \max\{C_i - d_i, 0\}$ ($C_i$ completion time of job $J_i$)

Example:

| Job | $J_1$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ | $J_6$ |
|---|---|---|---|---|---|---|
| Processing Time | 3 | 2 | 2 | 3 | 4 | 3 |
| Due date | 6 | 13 | 4 | 9 | 7 | 17 |
| Weight | 2 | 3 | 1 | 5 | 1 | 2 |

Sequence $\phi = J_3, J_1, J_5, J_4, J_1, J_6$

| Job | $J_3$ | $J_1$ | $J_5$ | $J_4$ | $J_2$ | $J_6$ |
|---|---|---|---|---|---|---|
| $C_i$ | 2 | 5 | 9 | 12 | 14 | 17 |
| $T_i$ | 0 | 0 | 2 | 3 | 1 | 0 |
| $w_i \cdot T_i$ | 0 | 0 | 2 | 15 | 3 | 0 |

# The Max Independent Set Problem

Also called “stable set problem” or “vertex packing problem”.
**Given:** an undirected graph $G(V, E)$ and a non-negative weight function $\omega$ on $V$ ($\omega : V \to \mathbf{R}$)

**Task:** A largest weight independent set of vertices, i.e., a subset $V' \subseteq V$ such that no two vertices in $V'$ are joined by an edge in $E$.

Related Problems:

### Vertex Cover

**Given:** an undirected graph $G(V, E)$ and a non-negative weight function $\omega$ on $V$ ($\omega : V \to \mathbf{R}$)
**Task:** A smallest weight vertex cover, i.e., a subset $V' \subseteq V$ such that each edge of $G$ has at least one endpoint in $V'$.

### Maximum Clique

**Given:** an undirected graph $G(V, E)$
**Task:** A maximum cardinality clique, i.e., a subset $V' \subseteq V$ such that every two vertices in $V'$ are joined by an edge in $E$

# Graph Partitioning

**Input:** A graph $G = (V, E)$, weights $w(v) \in Z^+$ for each $v \in V$ and $l(e) \in Z^+$ for each $e \in E$.
**Task:** Find a partition of $V$ into disjoint sets $V_1, V_2, \ldots, V_m$ such that $\sum_{v \in V_i} w(v) \leq K$ for $1 \leq i \leq m$ and such that if $E' \subseteq E$ is the set of edges that have their two endpoints in two different sets $V_i$, then $\sum_{e \in E'} l(e)$ is minimal.

Consider the specific case of graph bipartitioning, that is, the case $|V| = 2n$ and $K = n$ and $w(v) = 1, \forall v \in V$.

# Example: Scheduling in Parallel Machines

Total Weighted Completion Time on Unrelated Parallel Machines Problem

**Input:** A set of jobs $J$ to be processed on a set of parallel machines $M$. Each job $j \in J$ has a weight $w_j$ and processing time $p_{ij}$ that depends on the machine $i \in M$ on which it is processed.

**Task:** Find a schedule of the jobs on the machines such that the sum of weighted completion time of the jobs is minimal.

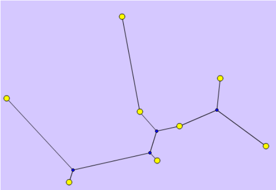
# Example: Steiner Tree

Steiner Tree Problem

**Input:** A graph $G = (V, E)$, a weight function $\omega : E \mapsto \mathbf{N}$, and a subset $U \subseteq V$.

**Task:** Find a Steiner tree, that is, a subtree $T = (V_T, E_T)$ of $G$ that includes all the vertices of $U$ and such that the sum of the weights of the edges in the subtree is minimal.

Vertices in $U$ are the special vertices and vertices in $S = V \setminus U$ are Steiner vertices.



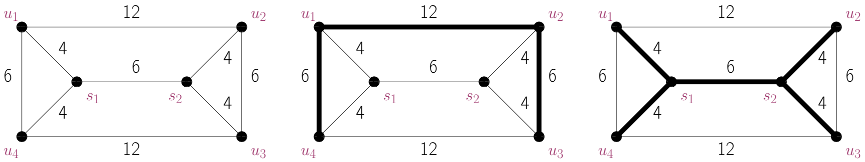



Figure: Vertices $u_1, u_2, u_3, u_4$ belong to the set $U$ of special vertices to be covered and vertices $s_1, s_2$ belong to the set $S$ of Steiner vertices. The Steiner tree in the second graph has cost 24 while the one in the third graph has cost 22.

1. Design one or more local search algorithms for the Steiner tree problem. In particular, define the solution representation and the neighborhood function.
2. Provide an analysis of the computational cost of the basic operations in the local search algorithms designed at the previous point. In particular, consider the size of the neighborhood, and the cost of evaluating a neighbor.