DM811

Heuristics for Combinatorial Optimization

Lecture 7
Local Search

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Course Overview

- ✔ Combinatorial Optimization, Methods and Models
- ✔ CH and LS: overview
- ✔ Working Environment and Solver Systems
- ✔ Methods for the Analysis of Experimental Results
- ✔ Construction Heuristics
- Local Search: Components, Basic Algorithms
- Local Search: Neighborhoods
- Efficient Local Search: Incremental Updates and Neighborhood Pruning
- Stochastic Local Search & Metaheuristics
- Configuration Tools: F-race
- Very Large Scale Neighborhoods

Examples: GCP, CSP, TSP, SAT, MaxIndSet, SMTWP, Steiner Tree, p-median, set covering

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Outline

Local Search Algorithms

Given a (combinatorial) optimization problem Π and one of its instances π:

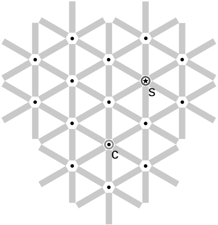
- search space $S(\pi)$
  specified by candidate solution representation:
  discrete structures: sequences, permutations, graphs, partitions
  (*e.g.*, for SAT: array, sequence of all truth assignments
  to propositional variables)

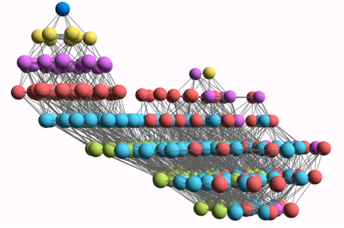
  Note: solution set $S'(\pi) \subseteq S(\pi)$
  (*e.g.*, for SAT: models of given formula)

- evaluation function $f_\pi : S(\pi) \rightarrow \mathbf{R}$
  (*e.g.*, for SAT: number of false clauses)

- neighborhood function, $\mathcal{N}_\pi : S \rightarrow 2^{S(\pi)}$
  (*e.g.*, for SAT: neighboring variable assignments differ
  in the truth value of exactly one variable)

Local search — global view





- vertices: candidate solutions (search positions)
- vertex labels: evaluation function
- edges: connect "neighboring" positions
- s: (optimal) solution
- c: current search position

Iterative Improvement

Iterative Improvement (II):
determine initial candidate solution s
while s has better neighbors **do**
  choose a neighbor s' of s such that $f(s') < f(s)$
  $s := s'$

- If more than one neighbor have better cost then need to choose one $\rightsquigarrow$ pivot rule
- The procedure ends in a local optimum $\hat{s}$:
  Def.: Local optimum $\hat{s}$ w.r.t. N if $f(\hat{s}) \leq f(s)\ \forall s \in N(\hat{s})$
- Issue: how to avoid getting trapped in bad local optima?
  - use more complex neighborhood functions
  - restart
  - allow non-improving moves

Local Search Algorithm

Further components [according to B4]

- set of memory states $M(\pi)$
  (may consist of a single state, for LS algorithms that do not use memory)

- initialization function $\texttt{init} : \emptyset \to S(\pi)$
  (can be seen as a probability distribution $\Pr(S(\pi) \times M(\pi))$ over initial search positions and memory states)

- step function $\texttt{step} : S(\pi) \times M(\pi) \to S(\pi) \times M(\pi)$
  (can be seen as a probability distribution $\Pr(S(\pi) \times M(\pi))$ over subsequent, neighboring search positions and memory states)

- termination predicate $\texttt{terminate} : S(\pi) \times M(\pi) \to \{\top, \bot\}$
  (determines the termination state for each search position and memory state)

Decision vs Minimization

LS-Decision(π)
input: problem instance $\pi \in \Pi$
output: solution $s \in S'(\pi)$ **or** $\emptyset$

$(s, m) := \texttt{init}(\pi)$

while not $\texttt{terminate}(\pi, \texttt{s}, \texttt{m})$ **do**
  $(s, m) := \texttt{step}(\pi, \texttt{s}, \texttt{m})$

if $s \in S'(\pi)$ **then**
  return s
else
  return $\emptyset$

LS-Minimization(π')
input: problem instance $\pi' \in \Pi'$
output: solution $s \in S'(\pi')$ **or** $\emptyset$

$(s, m) := \texttt{init}(\pi')$;
$s_b := s$;
while not $\texttt{terminate}(\pi', \texttt{s}, \texttt{m})$ **do**
  $(s, m) := \texttt{step}(\pi', \texttt{s}, \texttt{m})$;
  if $f(\pi', s) < f(\pi', \hat{s})$ **then**
    $s_b := s$;

if $s_b \in S'(\pi')$ **then**
  return s_b
else
  return $\emptyset$

Example: Uninformed random walk for SAT (1)

- **search space S:** set of all truth assignments to variables in given formula F (**solution set S':** set of all models of F)
- **neighborhood relation $\mathcal{N}$:** *1-flip neighborhood*, *i.e.*, assignments are neighbors under $\mathcal{N}$ iff they differ in the truth value of exactly one variable
- **evaluation function** not used, or $f(s) = 0$ if model $f(s) = 1$ otherwise
- **memory:** not used, *i.e.*, $M := \{0\}$

Example: Uninformed random walk for SAT (2)

- **initialization:** uniform random choice from S, *i.e.*, $\texttt{init}(, \{a', m\}) := 1/|S|$ for all assignments a' and memory states m
- **step function:** uniform random choice from current neighborhood, *i.e.*, $\texttt{step}(\{a, m\}, \{a', m\}) := 1/|N(a)|$ for all assignments a and memory states m, where $N(a) := \{a' \in S \mid \mathcal{N}(a, a')\}$ is the set of all neighbors of a.
- **termination:** when model is found, *i.e.*, $\texttt{terminate}(\{a, m\}, \{\top\}) := 1$ if a is a model of F, and 0 otherwise.

In Comet

Random Walk

queensLS0a.co

```
import cotls;
int n = 16;
range Size = 1..n;
UniformDistribution distr(Size);

Solver<LS> m();
var{int} queen[Size](m,Size) := distr.get();
ConstraintSystem<LS> S(m);

S.post(alldifferent(queen));
S.post(alldifferent(all(i in Size) queen[i] + i));
S.post(alldifferent(all(i in Size) queen[i] - i));
m.close();

int it = 0;
while (S.violations() > 0 && it < 50 * n) {
  select(q in Size, v in Size) {
    queen[q] := v;
    cout<<"chng @ "<<it<<": queen["<<q<<"]:="<<v<<" viol: "<<S.violations() <<endl;
  }
  it = it + 1;
}
cout << queen << endl;
```

In Comet

Another Random Walk

queensLS1.co

```
import cotls;
int n = 16;
range Size = 1..n;
UniformDistribution distr(Size);

Solver<LS> m();
var{int} queen[Size](m,Size) := distr.get();
ConstraintSystem<LS> S(m);

S.post(alldifferent(queen));
S.post(alldifferent(all(i in Size) queen[i] + i));
S.post(alldifferent(all(i in Size) queen[i] - i));
m.close();

int it = 0;
while (S.violations() > 0 && it < 50 * n) {
  select(q in Size : S.violations(queen[q])>0, v in Size) {
    queen[q] := v;
    cout<<"chng @ "<<it<<": queen["<<q<<"]:="<<v<<" viol: "<<S.violations()<<endl;
  }
  it = it + 1;
}
cout << queen << endl;
```

Summary: Local Search Algorithms
(as in [Hoos, Stützle, 2005])

For given problem instance π:

1. search space $S(\pi)$
2. neighborhood relation $\mathcal{N}(\pi) \subseteq S(\pi) \times S(\pi)$
3. evaluation function $f(\pi) : S \to \mathbf{R}$
4. set of memory states $M(\pi)$
5. initialization function $\texttt{init} : \emptyset \to S(\pi) \times M(\pi))$
6. step function $\texttt{step} : S(\pi) \times M(\pi) \to S(\pi) \times M(\pi)$
7. termination predicate $\texttt{terminate} : S(\pi) \times M(\pi) \to \{\top, \bot\}$