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September 22, 2006
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DM63 - Heuristics for Combinatorial Optimization Problems – Weekly Notes

Lecture 1, Fall 2006

Lecture September 21

We approached two other combinatorial optimization problems, the vertex coloring and the maximum satisfiability. For both we sketched possible construction heuristics.

We then introduced local search methods. They can be decomposed in three sub-procedures or under a different scheme in seven components. We focused on local search or perturbative search and in particular in the iterative improvement.

We defined through examples and speed-ups local search neighborhoods for the traveling salesman problem.

No competition tasks are posed this week.

In the next lecture, we will study the results of the Task 1 of the competition, design construction heuristics and local searches for other problems and introduce GRASP and Beam Search, two metaheuristics based on construction search. The next topic will be Variable Neighborhood Search.

Bibliographical Notes

The rest of the chapter of "Informed Search and Exploration" introduces local search methods and metaheuristics.

Local search for TSP is described in the rest of the article "Fast Algorithms for Geometric Traveling Salesman".

Web links referred during the lecture have been added in the Course Section Literature.

Exercises

Exercise 1

The Quadratic Assignment Problem (QAP) is the following:

- *Given:* n locations with a matrix $\mathbf{A} = [a_{ij}] \in \Re^{n\times n}$ of distances and n objects with a matrix $\mathbf{B} = [b_{kl}] \in \Re^{n\times n}$ of flows between them

- *Task:* Find the assignment φ of objects to locations that minimizes the sum of product between flows and distances, ie,
$$f(\varphi) = \sum b_{ij} a_{\varphi(i)\varphi(j)}$$

QAP Example:

$$\mathbf{A} = \begin{pmatrix} 0 & 4 & 3 & 2 & 1 \\ 4 & 0 & 3 & 2 & 1 \\ 3 & 3 & 0 & 2 & 1 \\ 2 & 2 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 0 & 2 & 3 & 4 \\ 2 & 2 & 0 & 3 & 4 \\ 3 & 3 & 3 & 0 & 4 \\ 4 & 4 & 4 & 4 & 0 \end{pmatrix}$$

The optimal solution is $\varphi = (1, 2, 3, 4, 5)$, that is, facility 1 is assigned to location 1, facility 2 is assigned to location 2, etc.

The value of $f(\varphi)$ is 100.

The QAP finds application in designing hospital layout and keyboard layout.
Design one or more construction heuristic for the QAP.

Exercise 2

Formalize the SUDOKU without preassignments as a vertex coloring problem. Consider then also preassignments and show how the graph G with precolored vertices (derived by the SUDOKU preassignments) can be transformed in an uncolored graph G' and the problem reduced to a classical vertex coloring.