Lecture September 17

The lecturer announced that the lecture on Monday October 1 is cancelled.
The lecture started by discussing the trade off between effectiveness and efficiency in the choice of neighborhood structure. It then continued focusing on local search for the TSP. In specific the following neighborhood structures were presented with examples at the black board: 2-opt, 2.5-opt, 3-opt, Or-opt. A basic code for the implementation of a first-improvement 2-opt local search was commented extensively, elaborating on the randomization of the examination and on the pruning of the neighborhood. Pruning might be exact or heuristic.

In the case of the TSP four pruning rules were explained at the black board: fixed-radius neighborhood search (exact), nearest neighborhood candidate list (heuristic), “don’t look bits” (heuristic). In addition it was stressed the importance of using efficient data structures.

An example showing how much involved the code of a well performing local search can become was given using the code by Stützle publically available.

The rest of the lecture consisted in the treatment of the other kind of most common neighborhoods: linear permutations, assignments and partitions/sets. Formal notation to describe each of these neighborhood is available on the slides.

An example for each of these representations was then treated at the black board. In particular, on the SMTWTP we stressed possible examination pruning for interchanges and the computational reduction in the examination of the insertion neighborhood made possible by a lexicographic order search. On the SAT problem we considered auxiliary data structures for the speed up of the delta evaluations in the flip neighborhood. On the max independent set we emphasized the existence of two alternative solution representations (fixed size - infeasible; variable size - feasible) and the implications they have in the solution strategy and in the definition of the evaluation function and neighborhood structure. (Some additional notes on this part might be added to the slides by the end of the week.)

The lecture finished with the following open problem for the students:

Consider an Iterative Best Improvement algorithm for solving the \( k \)-coloring problem under the approach \( k \)-fixed, complete improper colorings and one-exchange neighborhood. Let the evaluation function be defined by the number of violated constraints, ie, \( |E^c| \) where \( E^c = \{ uv : uv \in E \text{ and } \varphi(u) = \varphi(v) \} \) and \( \varphi \) a coloring. The computation of the quality of a given coloring takes \( O(|V|^2) \) and the complete examination of the neighborhood takes \( O(|V|^2k) \). How is it possible to reduce this last computation cost to \( O(|V|k) \)?

Bibliographical Notes

- The part on the local search for the TSP can be found in the text book on pages 372–377
- Further deepening on local search for TSP can be found in the articles by Johnson and McGeoch.; by Bentley; and by Fredman Johnson McGeoch and Ostheimer (full details in the Literature section).
- The part on the SAT problem can be found in the text book on pages 271–273
Scheduling problems are discussed on pages 417–432 of the text book. Though not yet treated that part constitute a necessary reading. Note that there is a difference in the terminology of the neighborhoods.