# Introduction to Haskell II 

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## Operators

Operators = built-in set of functions with short non-letter names.
Examples: + (addition), - (subtraction), ++ (list concatenation).
Most have two parameters and are written using infix notation:

$$
\begin{array}{ll}
2+3 & \leftarrow \text { infix } \\
\text { add } 23 & \leftarrow \text { usual prefix notation for functions }
\end{array}
$$

We can convert between "operator" and "standard" version of two parameter functions

$$
\begin{aligned}
& \text { Def: } \\
& \text { add } \mathrm{x} y=\mathrm{x}+\mathrm{y}
\end{aligned}
$$

| (+ |
| :---: |
|  |  |
|  |  |

## Associativity and Binding Power

To save on parentheses, operators (along with function application) are given diffent binding powers:

$$
2 * 3+f 4 \wedge 2=((2 * 3)+((f 4) \wedge 2))
$$

To resolve evaluation order of sequences of operators of equal binding power, they have an associativity assigned:

$$
\begin{aligned}
& 4+3+2+1=(((4+3)+2)+1) \\
& 4-3-2-1=(((4-3)-2)-1) \\
& 4 \wedge 3 \wedge 2 \wedge 1=(4 \wedge(3 \wedge(2 \wedge 1)))
\end{aligned}
$$

So + and - are left associative, whereas ^ is right associative.

## Do-it-yourself operators

You can define new operators (see Appendix C for rules).
Example: Minimum operator:

$$
\begin{aligned}
& \text { (??) : : Int }->\text { Int }->\text { Int } \\
& \text { x ?? y } \\
& \quad \mid \mathrm{x}>\mathrm{y}=\mathrm{y} \\
& \text { | otherwise }=\mathrm{x}
\end{aligned}
$$

Now:

$$
3 \text { ?? } 4 \leadsto 3
$$

Define associativiy and binding power:

$$
\text { infixl } 7 \text { ?? }
$$

## Pattern Matching

Definitions may use pattern matching on the parameters:

```
fac 0 = 1
fac n = fac (n-1) * n
fliptuple (x,y) = (y,x)
onAxe (0,y) = True
onAxe (x,0) = True
onAxe (x,y) = False
onAxe (0,_) = True
onAxe (_,0) = True
onAxe (_,_) = False
```


## Pattern Matching

A pattern is made of:

- Literals 24 , True, 's', []
- Identifiers $\mathrm{x}, \mathrm{y}$ (wild card _ is a nameless variable)
- Tuple constructor ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ )
- List constructor (x:xs)
- More constructors later...

A pattern can be hierarchical: ("hi", (x:(x):xs), $(2,0))$ )
A pattern can match or fail. To match, all sub-patterns must recursively match. When a match occurs, any matched identifiers are bound to the value matched.

## Polymorphism

## Types can be parametric

$$
\begin{aligned}
& \text { concat :: [[Int]] -> [Int] } \\
& \text { concat [] = [] } \\
& \text { concat (x:xs) = } x++ \text { concat } x s \\
& \text { concat }[[1,2],[4,5,6]] \sim[1,2,4,5,6] \\
& \text { concat :: [[a]] -> [a] } \\
& \text { concat [] = [] } \\
& \text { concat (x:xs) }=x++ \text { concat } x s \\
& \text { zip :: [a] -> [b] -> [(a,b)] } \\
& \text { zip (x:xs) (y:ys) = (x,y) : zip xs ys } \\
& \text { zip (x:xs) [] = [] } \\
& \text { zip [] zs = [] } \\
& \operatorname{zip}[1,2,3]\left[{ }^{\prime} a^{\prime}, ' b '\right] \sim[(1, ' a '),(2, ' b ')]
\end{aligned}
$$

## Functions as parameters and results

In Haskell, functions are values (value $\sim$ expression trees with empty leaves).
Can be passed to and from functions (then called high-order functions).
Very useful high-order functions:

```
map, filter, zipWith, foldl, foldr, foldl1, foldr1
map ::(a -> b) -> [a] -> [b]
map f [] = []
map f (x:xs) = f x : map f xs
```


## Functions as parameters and results

Generating functions as results:

- Composition:

$$
\begin{aligned}
& f=g \cdot h \\
& \text { twice } f=f . \quad f
\end{aligned}
$$

- Partial application (currying):

```
add :: Int -> Int -> Int
add x y = x + y
addOne :: Int -> Int
addOne = add 1 or
addOne = (1+)
addOneAll :: [Int] -> [Int]
addOneAll = map (add 1)
```


## Some Library Functions in Prelude

Check A Tour of the Haskell Prelude See
http://www.cs.uu.nl/~afie/haskell/tourofprelude.html

