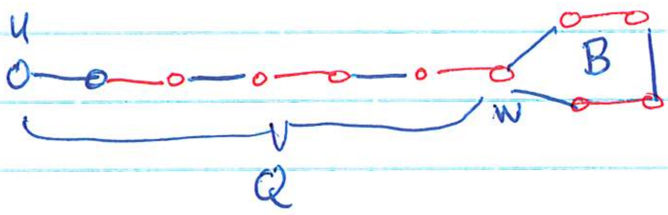
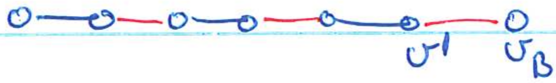
Theorem 10.4 in PS

$G=(V,E)$, M matching, B blossom found while searching from u.



G/B is obtained by contracting B to one vertex v_B:



M/B is M restricted to G/B

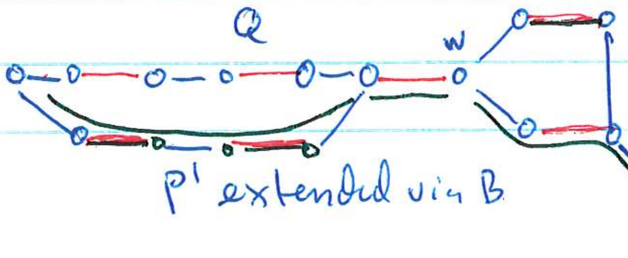
Then the following holds:

G has an M-augmenting path starting at u

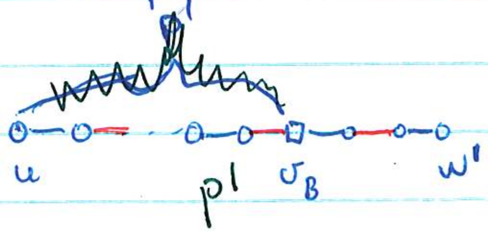
$\Updownarrow$

G/B has an M/B-augmenting path starting at u

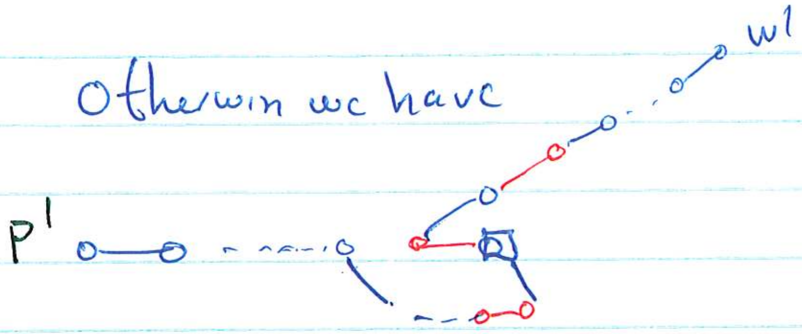
Proof: $\Uparrow$ let P' be an M/B-augm path in G/B starting in u
if P' does not contain v_B it is M-augm so
assume $v_B \in P'$ if P' uses the edge $v'v_B$ (see above)
then it is easy to extend P' to an M-augm path starting at u
in G



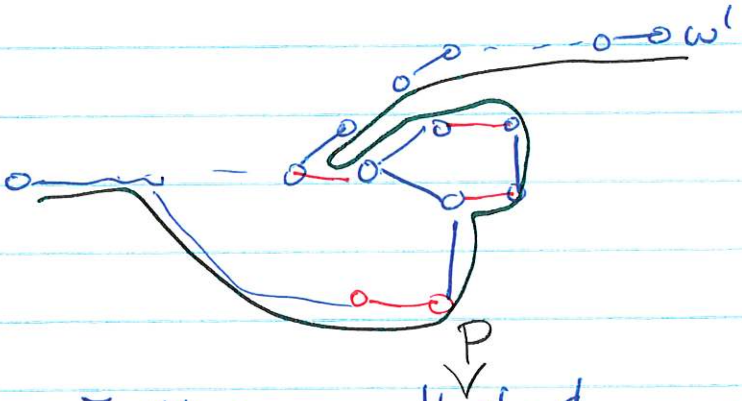
P' extended via B



Otherwise we have



and in G we get the path P:



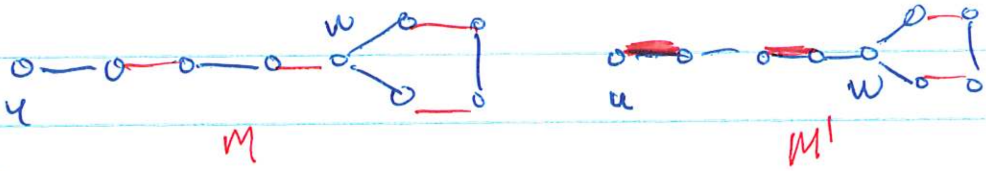
$\Downarrow$ $\exists$ M-augm path starting in u in G

$\Downarrow$ $\exists$ M^*-augm path starting in u in $G^* = P \cup Q \cup B$

where M^* is M restricted to G^*

$\Downarrow$ M^* not maximum in G^*

$\Downarrow$ M' not maximum in G^* $\quad M' = M \Delta Q$



$\Downarrow$ $\exists$ M'-augm path in G^* call it P'

$\Downarrow$ $\exists$ M'/B-augm path in G^*/B call it P''

(follow P' from the endpoint $v \neq w$ until it hits B for the first time)

$\Downarrow$

M'/B not maximum in G^*/B

$\Downarrow$

M/B not maximum in G^*/B

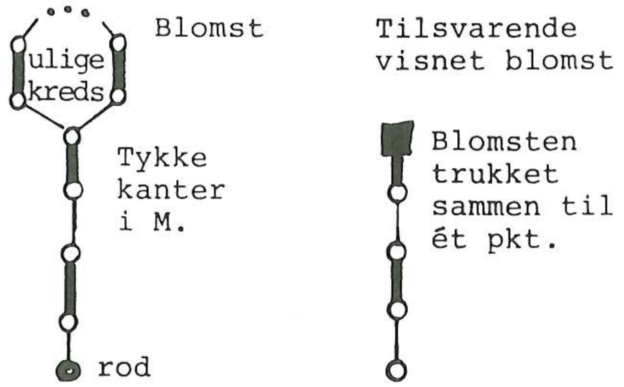
$\Downarrow$

$\exists$ M/B-augm path P_B in G^*/B

but G^* has only two unmatched vertices u and one more and the same holds for G^*/B so u is an end vertex of P_B $\square$

a la P&S

Edmond's algoritme for ikke-vægtet generel pardannelse (1965).



START
Graf G
Pardannelse M

Har G et nyt ikke-parret pkt. u ?

nej → M maximum i G. STOP!

ja ↓

Lad alt. træ T gro med det ikke-parrede pkt. u som rod

∃ blomst B

Lad B visne til punkt v_B. For $v \in B$ sættes blossom$(v) = v_B$. Lad T vokse videre

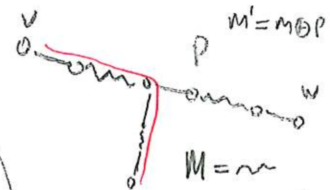
∃ alternerende vej

Lad evt. visnede blomster blomstre, og dan derved en ny pardannelse M i G

∄ alternerende vej

u betragtes som gammel

Hvis u ej er start på udv. vej mht M så aldrig senere:



Hvis ∃ udv. vej fra u mht $M' = M \oplus P$ så ∃ udv fra u mht original M

$O(V^3)$ for u enten parret eller gammelt:

max $O(V)$ blomster findes undervejs

$O(V^2)$ arbejdet at trække blomst sammen hhv slå en blomst op

selve søgning og udvidelse tager højst $O(V^2)$ for u

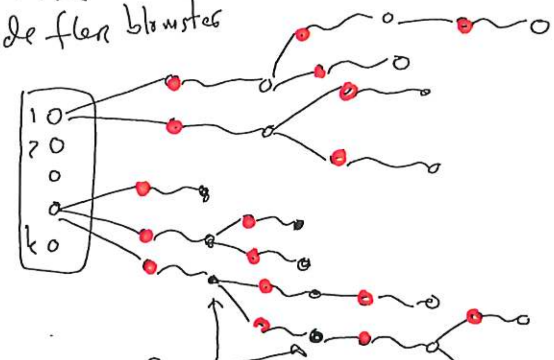
Sch Thm 5.1 (Tutte - Berge)

$\forall$ graph $G=(V,E)$ we have

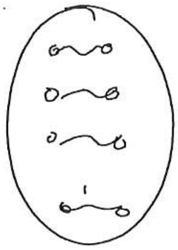
$$\nu(G) = \min_{U \subseteq V} \frac{1}{2}\left(|V| + |U| - \text{odd}(G-U)\right)$$

Tuttes sætn ud fra Edmonds alg.

kan ikke finde flere blomster



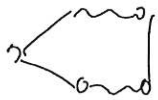
evt blomster



W

kan ikke nås

- altid ulige størrelse
- kun forbundet til • da ingen blomster



S = •

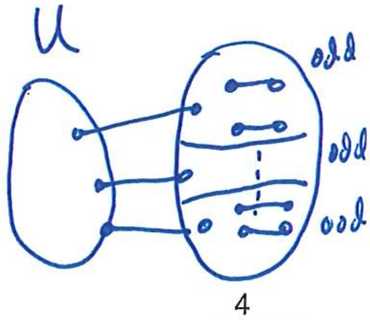
- ulige komponenter i G-S

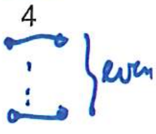
$\text{odd}(G-S) = |S| + k$

$$\nu(G) = |M| = \frac{n-k}{2} = \frac{1}{2}\left(n + |S| - \text{odd}(G-S)\right)$$

Clear that $\nu(G) \le \min \frac{1}{2}\left(|V| + |U| - \text{odd}(G-U)\right)$:

$$\begin{aligned} \nu(G) &\le |U| + \nu(G-U) \\ &\le |U| + \frac{1}{2}\left(|V-U| - \text{odd}(G-U)\right) \\ &= \frac{1}{2}\left(|V| + |U| - \text{odd}(G-U)\right) \end{aligned}$$





D

f-factors

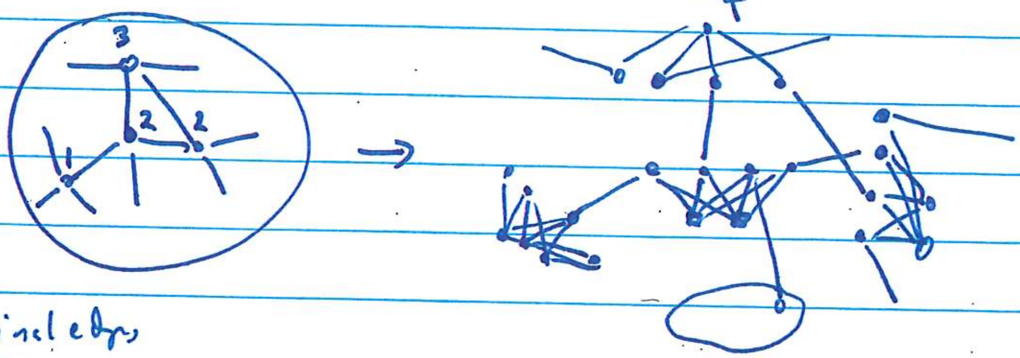
$G=(V,E) \qquad f: V \to \mathbb{Z}_+ \qquad f(v) \le d(v) \ \forall v$

Q: does $\exists\ H=(V,E')$ s.t. $d_H(v)=f(v)$ $E' \subseteq E$

$|A(v)| = d(v)$

$|B(v)| = d(v) - f(v)$

G $\qquad\qquad\qquad$ G_f



G_f has 1-factor

$\Updownarrow$

G has f-factor

$\boxed{\text{Reduction } G \to G_f \text{ is polynomial}}$

$$|V(G_f)| = \sum_{v \in V} (2d(v) - f(v)) \le 2\sum_v d(v) = 4|E|$$

$$|E(G_f)| = |E| + \sum_v d(v)(d(v)-f(v))$$

$$\le |E| + \sum_v d(v) \sum d(v)$$

$$\le 4|E|^2 + |E|$$