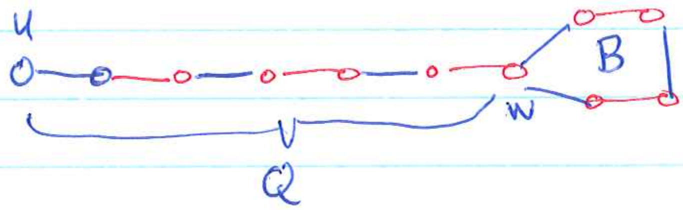
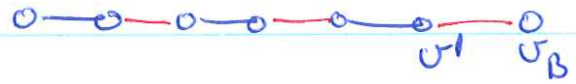
Theorem 10.4 in PS

$G=(V,E)$, M matching, B blossom found while searching from u.



G/B is obtained by contracting B to one vertex v_B:



M/B is M restricted to G/B

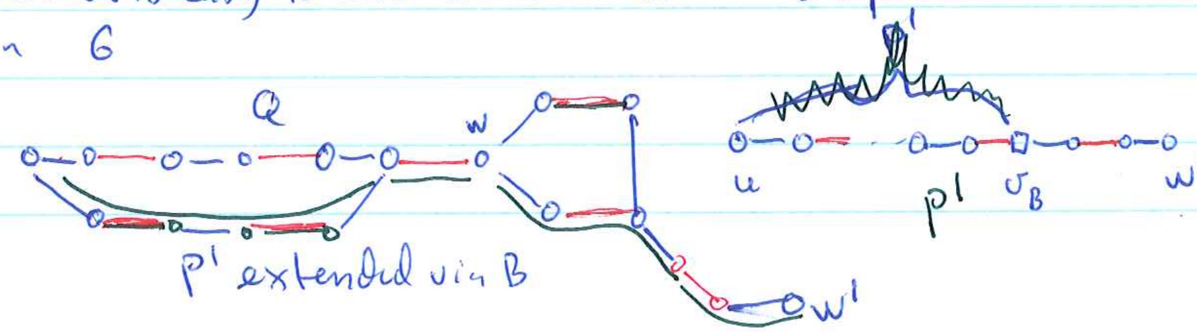
Then the following holds:

G has an M-augmenting path starting at u

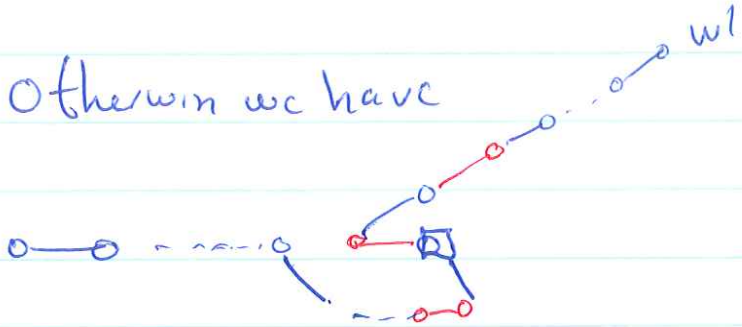
$\Updownarrow$

G/B has an M/B-augmenting path starting at u

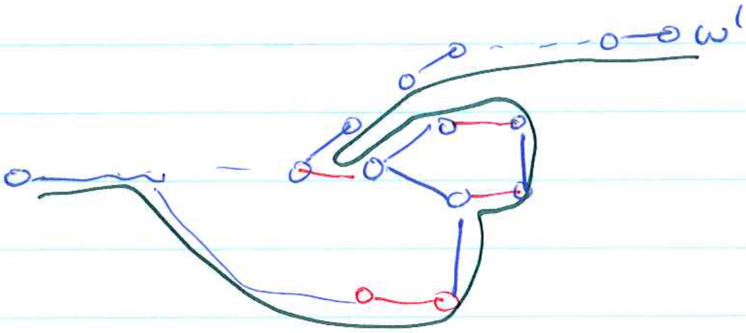
Proof: $\Uparrow$ Let P' be an M/B-augm path in G/B starting in u
if P' does not contain v_B it is M-augm so
assume $v_B \in P'$ if P' uses the edge $v'v_B$ (see above)
then it is easy to extend P' to an M-augm path starting at u
in G



Otherwise we have

P' 

and in G we get the path P:

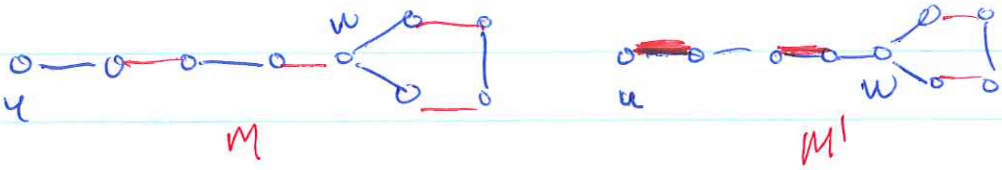


$\Downarrow$ $\exists$ M-augm path starting in u in G

$\Downarrow$ $\exists$ M^*-augm path starting in u in $G^* = P \cup Q \cup B$
where M^* is M restricted to G^*

$\Downarrow$ M^* not maximum in G^*

$\Downarrow$ M' not maximum in G^* $\quad M' = M \Delta Q$



$\Downarrow$ $\exists$ M'-augm path in G^* call it P'

$\Downarrow$ $\exists$ M'/B-augm path in G^*/B call it P''
(follow P' from ~~...~~ the endpoint $v \neq w$ until it hits B for the first time)

$\Downarrow$

M'/B not maximum in G^*/B

$\Downarrow$

M/B not maximum in G^*/B

$\Downarrow$

$\exists$ M/B-augm path P_B in G^*/B

but G^* has only two unmatched vertices u and one more and the same holds for G^*/B so u is an end vertex of P_B $\square$