Institut for Matematik og Datalogi
Syddansk Universitet

March 24, 2022
JBJ

DM867 – Fall 2022– Weekly Note 9

Stuff covered in week 12, 2022

- Matroid intersection and union. Korte and Vygen 13.5-13.6.

Midterm evaluation

Part of the class on March 28 will be spend on a midterm evaluation of the course. In order to prepare for this, think of (up to) 3 things you think work nicely with the course and (up to) 3 things that could be better. Then in class you write these statements on a piece of paper that I hand out. When this is done we pass your sheets around and when you receive a sheet that is not your own, you mark (with a 'x') those statements that you agree with.

Classes in Week 13

Material will be covered in the order as shown below.

- Midterm evaluation (hopefully much less than 1 hour)
- The Steiner tree problem. We will use Chapter 42 in the notes by Khuller on the home page
- The k-path problem for directed graphs. BJG Sections 9.1-9.2.

Exercises (March 30 or 31)

- Formulate the problem of deciding whether a digraph $D = (V, A)$ has a cycle factor, that is, a disjoint collection of directed cycles that cover each vertex of V as a matroid intersection problem. Hint: use one matroid to ensure in-degree at most 1 and one to ensure out-degree at most 1.
- Illustrate the matroid intersection algorithm for the matroids you defined above
  1. when D is the digraph with vertex set $\{1, 2, 3, 4, 5, 6\}$ and arc set $\{12, 34, 56, 13, 35, 51, 24, 46, 62\}$ and we start from the common independent set $X = \{12, 34, 56\}$
  2. when D is the same digraph as above but with the arc 46 missing and we start again from $X = \{12, 34, 56\}$.

- Suppose a digraph $D = (V, A)$ does not have a cycle factor. Show that this happens if and only if there exists a partition $A = A_1 \cup A_2$ such that the number of vertices with in-degree at least one wrt A_1 plus the number of vertices that have out-degree at least one wrt A_2 is less than $|V|$.
- Show how to formulate the problem of deciding whether a graph $G = (V, E)$ has a pair of edge-disjoint spanning trees as a matroid intersection problem.