Backbone colorings along spanning trees, spanning paths and perfect matchings

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Abstract

Given a graph $G = (V, E)$ and a spanning subgraph H of G (the backbone of G), a backbone coloring for G and H is a proper vertex coloring $V \to \{1, 2, \ldots\}$ of G in which the colors assigned to adjacent vertices in H differ by at least two. In this talk we discuss the cases where the backbone is either a soanning tree, a spanning path, or a perfect matching. We determine that for these backbones of G the number of colors needed for a backbone coloring of G can roughly differ by a multiplicative factor of at most 2, $\frac{3}{2}$ and $\frac{4}{3}$, respectively, from the chromatic number $\chi(G)$. We also briefly discuss the computational complexity of the problem "Given a graph G with a backbone H, and an integer ℓ, is there a backbone coloring for G and H with at most ℓ colors?"; it jumps from polynomial to NP-complete between $\ell = 4$ and $\ell = 5$ for spanning trees (paths) and between $\ell = 3$ and $\ell = 4$ for perfect matchings. Finally, we consider the case where G is a planar graph, and discuss some open problems.