Color-critical graphs and hypergraphs with few edges

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Abstract

Consider a graph G and assign to every vertex x of G a set $L(x)$ of colors. Such an assignment L of sets to vertices in G is referred to as a *color scheme* or briefly as a *list* for G. An *L-colouring* of G is a mapping c of $V(G)$ into the set of colors such that $c(x) \in L(x)$ for all $x \in V(G)$ and $c(x) \neq c(y)$ whenever $xy \in E(G)$. If G admits an L-coloring, then G is also called L-colorable.

We say that G is *L-critical* if G is not L-colorable but every proper subgraph of G is L-colorable. In case of $|L(x)| = k-1$ for all $x \in V(G)$ we also use the term *k-list-critical* and in case of $L(x) = \{1, \ldots, k-1\}$ for all $x \in V(G)$ we also use the term *k-critical.* Clearly, a graph G is k-critical if and only if $\chi(H) < \chi(G) = k$ for every proper subgraph H of G.

Critical graphs were first defined and studied by Dirac around 1950. As an extension of Brooks' theorem Dirac proved in 1957 that if $G = (V, E)$ is a k-critical graph with $k \geq 4$ and $G \neq K_k$, then

$$2|E| \geq (k-1)|V| + (k-3).$$

In the talk we present some new lower bounds for the number of edges of k-critical respectively k-list-critical graphs and hypergraphs. These bounds improve earlier bounds established by Dirac, Gallai, Krivelevich, Burstein, Lovász and Woodall.