# Convex Extensions of Set Functions

Jack Edmonds

**Abstract**

Besides, submodular set functions, there are many other interesting subadditive set functions, say $f(S)$ of the subsets $S$ of a set $V$. Let polytope $P = \{x : x \geq 0; x(S) \leq f(S)$ for every S $\}$. In various fields it has sometimes become popular to liken submodularity to convexity because $f(c) = \max\{cx : x \in P\}$, expressed by a formula for the dual of this LP which follows from the greedy algorithm for polymatroids, is a convex extension of the submodular set function $f(S)$.

Less recognized is the fact that for any subadditve set function $f(S)$, $f(c)$ is a convex extension of $f(S)$, for example where $V$ is the node set of a graph $G$ and $f(S) = max\{|J| : J$ is a subset of $S$ which is stable in G$\}$.

Where $G$ is perfect or a line graph, the extension $f(c)$ is especially nice because then $P$ is the convex hull of the stable sets of $G$, and like for submodularity, there are nice formulas for the extension $f(c)$.

For what other graph classes is $P$ the convex hull of the stable sets?