Rigidity of Graphs and Frameworks

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Abstract

The first reference to the rigidity of frameworks in the mathematical literature occurs in a problem posed by Euler in 1776. Consider a polyhedron P in 3-space. We view P as a ' panel-and-hinge framework' in which the faces are 2-dimensional panels and the edges are 1-dimensional hinges. The panels are free to move continuously in 3-space, subject to the constraints that the shapes of the panels and the adjacencies between them are preserved, and that the relative motion between pairs of adjacent panels is a rotation about their common hinge. The polyhedron P is rigid if every such motion results in a polyhedron which is congruent to P. Euler's conjecture was that every polyhedron is rigid.

The conjecture was verified for the case when P is convex by Cauchy in 1813. Gluck showed in 1975 that it is true when P is 'generic' i.e. there are no algebraic dependencies between the coordinates of the vertices of P. Connelly finally disproved the conjecture in 1982 by constructing a polyhedron which is not rigid.

I will describe results and open problems concerning the rigidity of various other types of frameworks. I will be mostly concerned with the generic case for which the problem of characterizing rigidity reduces to a problem which depends only on the incidence graph of the framework.

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