Doubly-critical hypergraphs

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A graph G is *doubly-critical* if the removal of any two adjacent vertices of G reduces the chromatic number by two. Erdős and Lovász conjectured that the only connected doubly-critical graphs are the complete graphs — equivalently, that every doubly-critical graph is the union of a complete graph and a set of isolated vertices.

We examine a natural extension of the problem to hypergraphs. Let us say that a hypergraph H is doubly-critical if the chromatic number drops by two whenever we remove the vertex set of any hyperedge, along with all hyperedges intersecting it. We construct a number of examples suggesting that there may be no characterisation of doubly-critical hypergraphs as simple as the one conjectured for graphs. On the positive side, we show that a simple characterisation does exist in the special case of hypergraphs consisting of all circuits of a matroid. Joint work with Matěj Stehlík and Riste Škrekovski.