THE DIRECTED GRID THEOREM

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Abstract. The grid theorem, originally proved by Robertson and Seymour in Graph Minors V in 1986, is one of the most central results in the study of graph minors. It has found numerous applications in algorithmic graph structure theory, for instance in bidimensionality theory, and it is the basis for several other structure theorems developed in the graph minors project.

In the mid-90s, Reed and Johnson, Robertson, Seymour and Thomas, independently, conjectured an analogous theorem for directed graphs, i.e. the existence of a function $f : \mathbb{N} \to \mathbb{N}$ such that every digraph of directed tree-width at least $f(k)$ contains a directed grid of order k. In an unpublished manuscript from 2001, Johnson, Robertson, Seymour and Thomas gave a proof of this conjecture for planar digraphs. But for over a decade, this was the most general case proved for the Reed, Johnson, Robertson, Seymour and Thomas conjecture.

In this paper, nearly two decades after the conjecture was made, we are finally able to confirm the Reed, Johnson, Robertson, Seymour and Thomas conjecture in full generality and to prove the directed grid theorem.

As consequence of our results we are able to improve the result in Reed et al. in 1996 on disjoint cycles of length at least l. We also show how to relate this directed grid theorem to the half-integral disjoint paths. We expect many more algorithmic results to follow from the grid theorem.

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