A POLYNOMIAL ALGORITHM FOR THE 2-PATH PROBLEM FOR SEMICOMPLETE DIGRAPHS*

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1992, SIAM J. DISC. MATH

k-linked: (k-path)

Semicomplete digraph:

THEOREM 5.1. *There exists a polynomial algorithm for the following problem for semicomplete digraphs: Let T be a semicomplete digraph and x_1, x_2, y_1, y_2 be four different vertices of T. Decide whether T has a pair of disjoint (x_1, y_1)-, (x_2, y_2)-paths.*

(1989, Bang-Jensen, Annals of Discrete Mathematics)

THEOREM 2.1 (see [3]). *Let T be a semicomplete digraph and let x_1, x_2, y_1, y_2 be distinct vertices of T. If $T - \{x_i, y_i\}$ has three internally disjoint (x_{3-i}, y_{3-i})-paths and $T - \{x_{3-i}, y_{3-i}\}$ has two internally disjoint (x_i, y_i)-paths, for $i = 1$ or 2, then T has a pair of disjoint (x_1, y_1)-, (x_2, y_2)-paths.*

History

(1970，伦敦数学会会刊)

Theorem 3.5.2. (Jung 1970; Larman & Mani 1970)
There is a function $f:\mathbb{N}\to\mathbb{N}$ such that every $f(k)$-connected graph is k-linked, for all $k\in\mathbb{N}$.

- $f(k)=3n\cdot 2^{\binom{3n}{2}}$.
- $22k \to 16k \to 12k \to 10k$ (2005, Thomas, Wollan)

↓ natural generalization digraph?

(1980, Thomassen. 欧洲组合)

Conjecture 1.1. [19] *There exists a function $f(k)$ such that every $f(k)$-strong digraph is k-linked.*

↓ 构造了一类反例图

(1991, Thomassen, combinatorica)

Theorem 1. *For each natural number k, there exists a non-2-linked strongly k-connected digraph D_k.*

↓ for special digraph classes

(1989, Bang-Jensen, Annals of Discrete Mathematics)

best possible

Corollary 10.5.2 [859] *Every 5-strong semicomplete digraph is 2-linked.*

Polynomial for semicomplete.

(本文, 1992, Bang-Jensen and Thomassen, SIAM J. Disc. MATH)

THEOREM 5.1. *There exists a polynomial algorithm for the following problem for semicomplete digraphs: Let T be a semicomplete digraph and x_1, x_2, y_1, y_2 be four different vertices of T. Decide whether T has a pair of disjoint (x_1, y_1)-, (x_2, y_2)-paths.*

local semicomplete digraph:

quasi-transitive digraph:

(1999, Bang-Jensen, Discrete Mathmatics)

Conjecture 3.9. Every 5-strong locally semicomplete digraph is 2-linked.

Corollary 4.6. *Every 5-strong quasi-transitive digraph is 2-linked.*

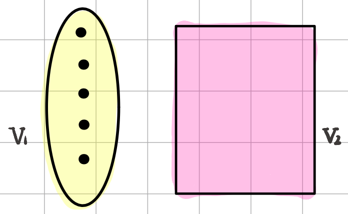
Theorem 4.7. *There exists a polynomial algorithm for the 2-linkage problem for quasi-transitive digraphs.*

(2016, Bang Jensen and Christiansen, J. Graph Theory)

Theorem 4.4. *For every fixed k, there exists a polynomial algorithm to solve the k-linkage problem on locally semicomplete digraphs.*

Theorem 6.1. *Let D be a 5-strong locally semicomplete digraph. Then D is 2-linked.*

split digraph:



$D=(V_1, V_2; A)$

(2024, Bang-Jensen and Yun Wang, J. Graph Theory)

Problem 3. Is there a polynomial algorithm for the 2-linkage problem for split digraphs?

Problem 4. Is every 6-strong split digraph 2-linked?

(2026)

Theorem 1.2. *Every 6-strong split digraph is 2-linked.*

Theorem 1.5. *Every 5-strong semicomplete split digraph is 2-linked.* (tight)

Theorem 1.7. *Every 6-strong semicomplete multipartite digraph is 2-linked.*

Main results:

THEOREM 5.1. *There exists a polynomial algorithm for the following problem for semicomplete digraphs: Let T be a semicomplete digraph and x_1, x_2, y_1, y_2 be four different vertices of T. Decide whether T has a pair of disjoint (x_1, y_1)-, (x_2, y_2)-paths.*

Open problem:

(2024, Bang-Jensen and Yun Wang, J. Graph Theory)

Problem 3. Is there a polynomial algorithm for the 2-linkage problem for split digraphs?

△

THEOREM 5.1. *There exists a polynomial algorithm for the following problem for semicomplete digraphs: Let T be a semicomplete digraph and x_1, x_2, y_1, y_2 be four different vertices of T. Decide whether T has a pair of disjoint (x_1, y_1)-, (x_2, y_2)-paths.*

call it desired path

Sketch of the proof.

- there is an (x_i, y_i)-path in $T-\{x_{3-i}, y_{3-i}\}$ for $i=1,2$. → 否则, return no
- T is strong and T contains none of the arcs $x_1 \to y_1$, $x_2 \to y_2$. → 否则, return Yes

- From theorem 2.1:

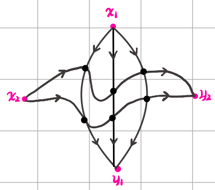
  if $T-\{x_i, y_i\}$ has three internally disjoint (x_{3-i}, y_{3-i})-paths

  and $T-\{x_{3-i}, y_{3-i}\}$ has two internally disjoint (x_i, y_i)-paths for i=1 or 2

  $\Downarrow$

  T has desired paths. ✓ (2,3) ✓

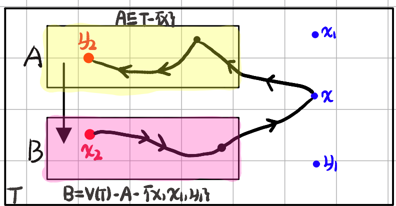
  (1,1), (1,2), (2,2) ?



Case 1. The local connectivity from x_i to y_i in $T-\{x_{3-i}, y_{3-i}\}$ is 1 for i=1 or 2.

↓ take i=2

- By Menger theorem, there exist separator $x \in T-\{x_1, y_1\}$ such that there is no (x_2, y_2)-path in $T-\{x, x_1, y_1\}$.



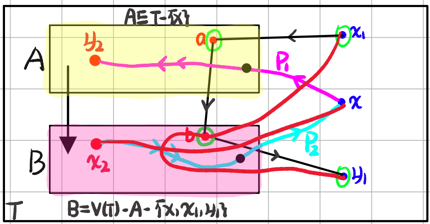
A

B

$A \mapsto B$

(1) If A contains a vertex a dominated by x_1 such that $T-\{x_1,y_1,a\}$ has an (x,y_2)-path P_1.

and B contains a vertex b dominating y_1 such that $T-\{x_1,y_1,b\}$ has an (x_2,x)-path P_2.

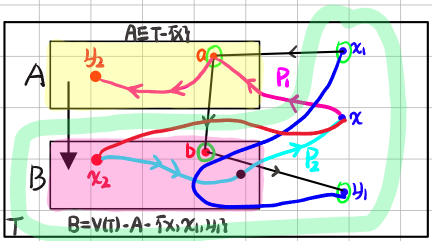


$x_1 \to a \to b \to y_2$ and $P_1 \cup P_2$

$\Downarrow$

T has desired paths. ✓

(2) Suppose that a does not exist. We reduce the problem to a smaller one.



T has desired paths ✓

$\Updownarrow$

$T[B \cup \{x, x_1, y_1\}]$ contains a pair of disjoint (x_1,y_1)-, (x_2,x)-paths.

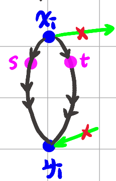
(1,1), (1,2), (2,2)

Case 2. There exist two internally disjoint (x_i, y_i)-paths in $T - \{x_{3-i}, y_{3-i}\}$ for i=1, 2.

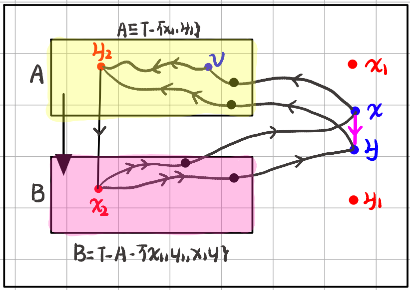
(1) If all 2-separator of x_1, y_1 in $T-\{x_2,y_2\}$ and all 2-separator of x_2, y_2 in $T-\{x_1,y_1\}$ are trivial.

$\Downarrow$ Theorem 4.1

T has desired paths ✓



(2) Let $\{x,y\}$ be a nontrivial 2-separator of x_2, y_2 in $T-\{x_1,y_1\}$



⇓

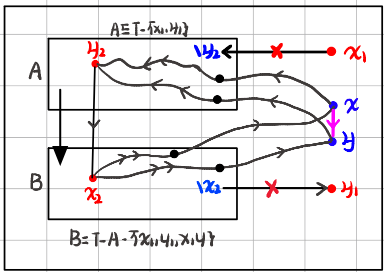
define A, B, $|A|,|B|\geq 2$

$A \mapsto B$

$x \to y$ (without loss of generality)

$\Rightarrow |A|,|B|\geq 2$

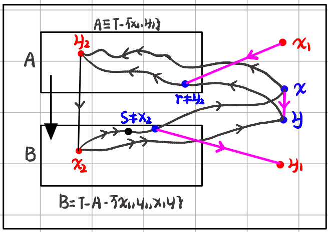
Subcase 2.1. There are no arcs from x_1 to $A-y_2$, or there are no arcs from $B-x_2$ to y_1.



- T has desired paths ✓
- reduce the problem to a smaller one T'. ✓ $T \to T'$(smaller)

resulting semicomplete digraph T' has desired paths if and only if T also has them

Subcase 2.2. $x_1 \rightarrow r$ for some $r \in A - y_2$, and $s \rightarrow y_1$ for some $s \in B - x_2$.



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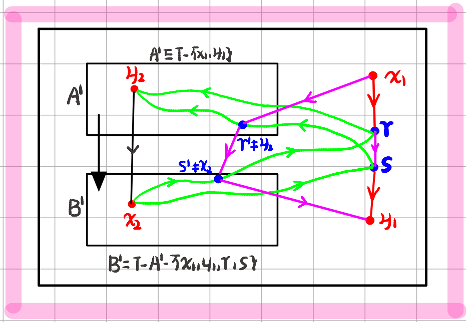


- T has desired paths ✓

- T has desired paths if and only if

at least one of $T-\{x_2, s, r', y_2\}$, $T-\{x_2, s', r, y_2\}$ contains an (x_1,y_1)-path.

↑

(BFS|DFS)

The complexity of the algorithm: $O(n^5)$

- the analysis of all separating set of size 2 take $O(n^4)$ time.

- flow calculation and all other action at most $O(n^3)$ time.

- subcase 2.1 (Lemma 4.2) take $O(n^5)$ time. $\square$

tools

THEOREM 2.1 (see [3]). *Let T be a semicomplete digraph and let x_1, x_2, y_1, y_2 be distinct vertices of T. If $T - \{x_i, y_i\}$ has three internally disjoint (x_{3-i}, y_{3-i})-paths and $T - \{x_{3-i}, y_{3-i}\}$ has two internally disjoint (x_i, y_i)-paths, for $i = 1$ or 2, then T has a pair of disjoint (x_1, y_1)-, (x_2, y_2)-paths.*

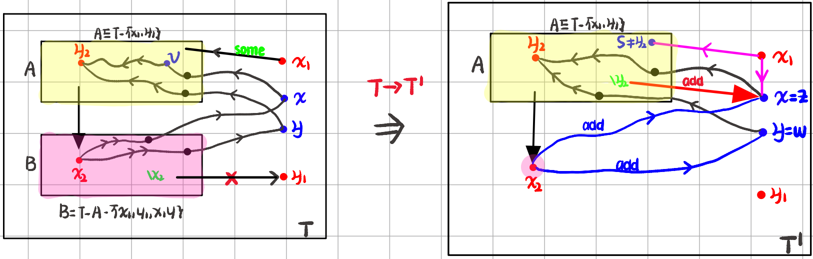
THEOREM 4.1. *Let T be a semicomplete digraph, and let x_1, x_2, y_1, y_2 be distinct vertices of T such that, for each $i = 1, 2$, there are two, but not three, internally disjoint (x_i, y_i)- paths in $T - \{x_{3-i}, y_{3-i}\}$. Suppose that all (x_i, y_i)-separators of size 2 in $T - \{x_{3-i}, y_{3-i}\}$ are trivial, for $i = 1, 2$. Then T has a pair of disjoint $(x_1, y_1)-, (x_2, y_2)$-paths.*

LEMMA 4.2. *Let T be a semicomplete digraph, and let x_1, x_2, y_1, y_2 be distinct vertices such that there are two internally disjoint (x_2, y_2)-paths in $T - \{x_1, y_1\}$. Suppose that there exists a nontrivial 2-separator $\{x, y\}$ of x_2 and y_2 in $T - \{x_1, y_1\}$ such that there is no arc from $B - x_2$ to y_1, where A and B form any patition of $T - \{x_1, y_1, x, y\}$ such that $x_2 \in B$, $y_2 \in A$, and all arcs between A and B go from A to B.*

Transform T into a new semicomplete digraph T' as follows:

1. *If x_1 dominates some vertex in $A - y_2$ then*
   - *If there exists a vertex $b \in B - x_2$ such that $b \rightarrow x$ and there is an (x_2, y)-path in $T(B \cup \{y\} \setminus \{b\})$, then add all arcs from $A - y_2$ to x that are not present already.*
   - *If there exists a vertex $b \in B - x_2$ such that $b \rightarrow y$ and there is an (x_2, x)-path in $T(B \cup \{x\} \setminus \{b\})$, then add all arcs from $A - y_2$ to y that are not present already.*
2. *Add the arcs $x_2 \rightarrow x, x_2 \rightarrow y$ if they are not present already;*
3. *Add the arc $x_1 \rightarrow z$ for $\{z, w\} = \{x, y\}$ if $T(B \cup \{z, w, x_1\})$ has a pair of disjoint $(x_1, z)-, (x_2, w)$-paths, and the arc $x_1 \rightarrow z$ is not present already;*
4. *Delete the vertices of $B - x_2$.*

Call the added arcs special arcs. Then the resulting semicomplete digraph T' has disjoint (x_1, y_1)-, (x_2, y_2)-paths if and only if T also has them.



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proof. From theorem 2.1:

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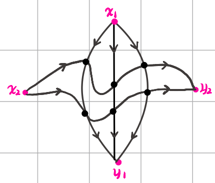
⇓

T has desired paths. ✓ (2,3) ✓



(1,1), (1,2), (2,2) ?

(1,1), (1,2)?

Case 1. The local connectivity from x_i to y_i in $T - \{x_{3-i}, y_{3-i}\}$ is 1 for i=1 or 2.

↓ take $i=2$

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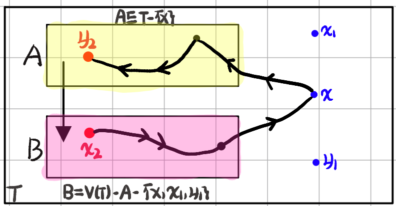

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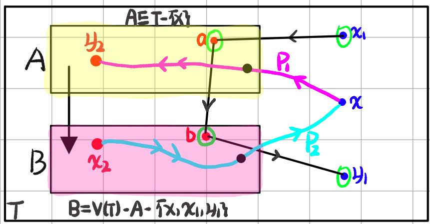
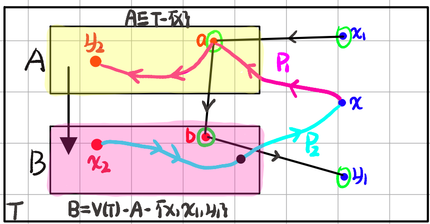


$x_1 \to a \to b \to y_2$ and $P_1 \cup P_2$

$\Downarrow$

T has desired paths. ✓

(2) Suppose that a does not exist. We reduce the problem to a smaller one.



T has desired paths ✓

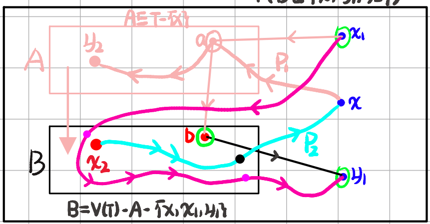
$\Updownarrow$

$T(B\cup\{x,x_1,y_1\})$ contains a pair of disjoint (x_1,y_1)-, (x_2,x)-paths.

↑ 不斷地 iterate.

$\Updownarrow$

$T(B\cup\{x,y_1,y_2\})$



(1,1), (1,2), (2,2)

12.21

Case 2. There exist two internally disjoint (x_i, y_i)-paths in $T - \{x_{3-i}, y_{3-i}\}$ for i=1, 2.

(1) If all 2-separator of x_1, y_1 in $T-\{x_2, y_2\}$ and all 2-separator of x_2, y_2 in $T-\{x_1, y_1\}$ are trivial.

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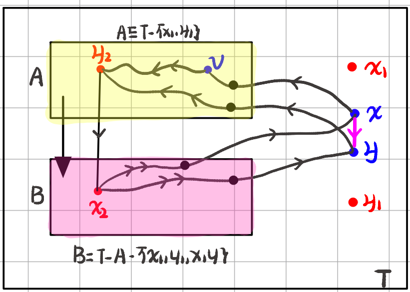
$\Downarrow$

T has desired paths ✓

(2) Let $\{x, y\}$ be a nontrivial 2-separator of x_2, y_2 in $T-\{x_1, y_1\}$



$\Downarrow$

define A, B, $|A|, |B| \geq 2$

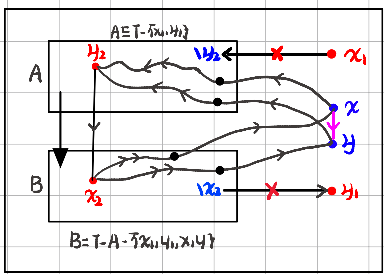
A

B

$A \mapsto B$

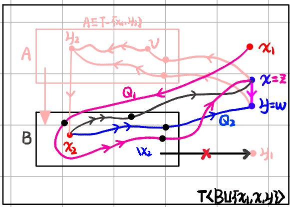
$x \to y$ (without loss of generality)

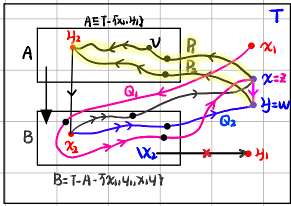
Subcase 2.1. There are no arcs from x_1 to $A - y_2$, or there are no arcs from $B - x_2$ to y_1.



or

- whether there exist disjoint (x_1,z)-, (x_2,w)- paths in $T\langle B\cup\{x_1,x,y\}\rangle$, where $\{z,w\}=\{x,y\}$ (using the flow version, we can actually find these path if they exist).





(1) If $T\langle B\cup\{x_1,x,y\}\rangle$ also has these paths $\Rightarrow$ the desired paths exist.

no arc from $B-x_2$ to y_1

$\Downarrow$

- there are two internally disjoint paths P_1, P_2 from $\{x,y\}$ to y_2 in $T\langle A\cup\{x,y\}\rangle$.
- Also, $T-\{x_2,y_2\}$ has a path P from x_1 to y_1. no arc $B\backslash x_2\to y_1$
- Going backward on P until we meet a vertex in one of the P_1, P_2. (P必定与P_1,P_2相交)

$\downarrow$
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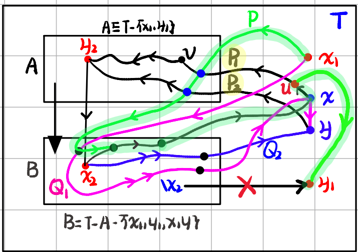
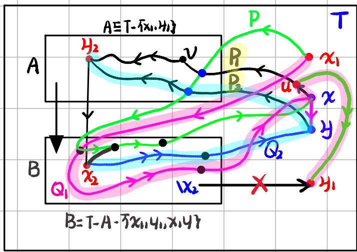


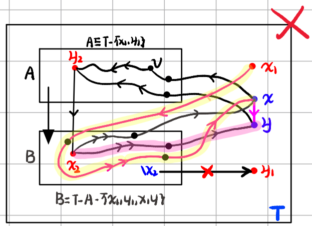
$\Rightarrow$



(2) $T(B \cup \{x_1, x, y\})$ does not have both pairs of path.

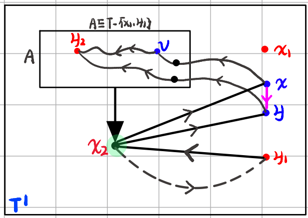
$\Downarrow$

T: contract B to $\{x_2\}$ $\Rightarrow$ new semicomplete digraph T' ($|T'|<|T|$) ($|A|,|B|\geq 2$)



$T \to T'$

$\Rightarrow$



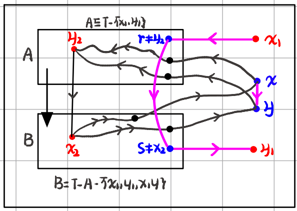
LEMMA 4.2. *Let T be a semicomplete digraph, and let x_1, x_2, y_1, y_2 be distinct vertices such that there are two internally disjoint (x_2, y_2)-paths in $T - \{x_1, y_1\}$. Suppose that there exists a nontrivial 2-separator $\{x, y\}$ of x_2 and y_2 in $T - \{x_1, y_1\}$ such that there is no arc from $B - x_2$ to y_1, where A and B form any patition of $T - \{x_1, y_1, x, y\}$ such that $x_2 \in B$, $y_2 \in A$, and all arcs between A and B go from A to B.*

Transform T into a new semicomplete digraph T' as follows:

1. *If x_1 dominates some vertex in $A - y_2$ then*
   - *If there exists a vertex $b \in B - x_2$ such that $b \to x$ and there is an (x_2, y)-path in $T(B \cup \{y\} \setminus \{b\})$, then add all arcs from $A - y_2$ to x that are not present already.*
   - *If there exists a vertex $b \in B - x_2$ such that $b \to y$ and there is an (x_2, x)-path in $T(B \cup \{x\} \setminus \{b\})$, then add all arcs from $A - y_2$ to y that are not present already.*
2. *Add the arcs $x_2 \to x, x_2 \to y$ if they are not present already;*
3. *Add the arc $x_1 \to z$ for $\{z, w\} = \{x, y\}$ if $T(B \cup \{z, w, x_1\})$ has a pair of disjoint (x_1, z)–, (x_2, w)-paths, and the arc $x_1 \to z$ is not present already;*
4. *Delete the vertices of $B - x_2$.*

Call the added arcs special arcs. Then the resulting semicomplete digraph T' has disjoint (x_1, y_1)-, (x_2, y_2)-paths if and only if T also has them.

Subcase 2.2. $x_1 \to r$ for some $r \in A - y_2$, and $s \to y_1$ for some $s \in B - x_2$.

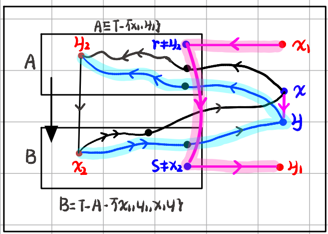


$x, y \notin A \cup B$

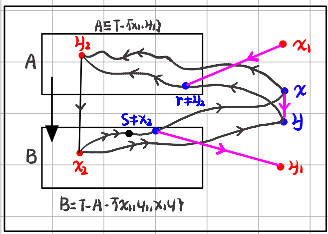
(1) Then T contains the path $x_1 \to r \to s \to y_1$. If $T - \{x_1, r, s, y_1\}$ has an (x_2, y_2)-path,

$\Downarrow$

T has the desired paths.



$\Rightarrow$



(2) Hence we may assume that this is not the case.

$\Downarrow$

$\{r, s\}$ must separate x_2 from y_2 in $T - \{x_1, y_1\}$.

$\Downarrow$

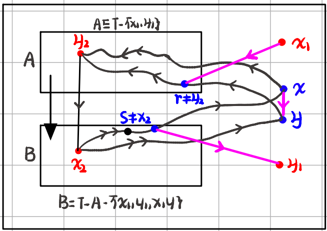
$\{r, s\}$ is also a nontrivial 2-separator of x_2 and y_2 in $T - \{x_1, y_1\}$

$\Downarrow$ 最开始定义$\{x, y\}$是non-trivial 2-separator of x_2 and y_2 in $T - \{x_1, y_1\}$

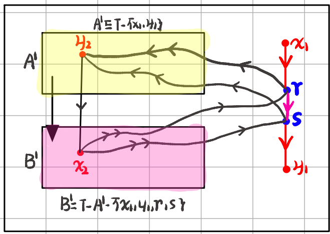
$\Downarrow$

现在替换为$\{r, s\}$是non-trivial 2-separator of x_2 and y_2 in $T - \{x_1, y_1\}$

Update: $\{x, y\} \longrightarrow \{r, s\}$, $A \rightarrow A'$, $B \rightarrow B'$



$\Rightarrow$



① $x_1 \to r$, $s \to y_1$, $r \in A - y_2$, $s \in B - x_2$, $r \to s$

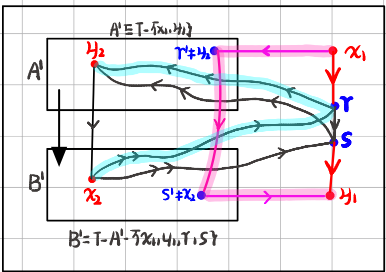
- If there is no arc from $x_1 \to A' - y_2$ or no arc from $B' - x_2 \to y_1$.

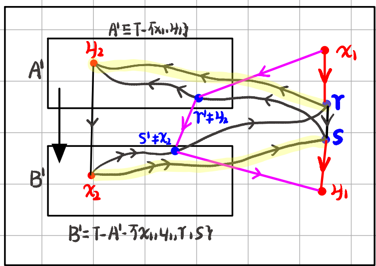
$\Big\downarrow$ subcase 2.1

① desired paths exist. ② reduce smaller one.

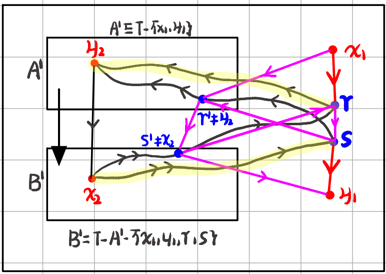
$\Downarrow$ subcase 2.2

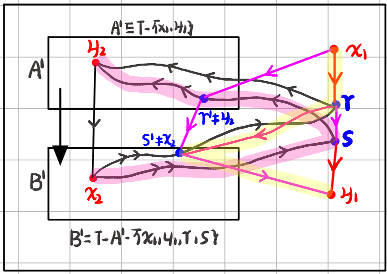
- we have some $r' \in A' - y_2$ and some $s' \in B' - x_2$ such that $x_1 \to r' \to s' \to y_1$ in T.





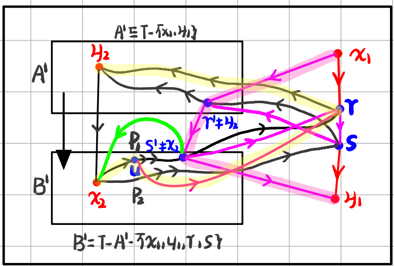
• If $r \to s'$ or $r' \to s$. $\Rightarrow$ T contains the desired paths

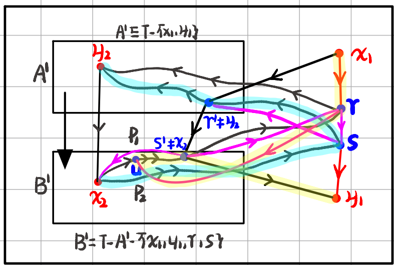


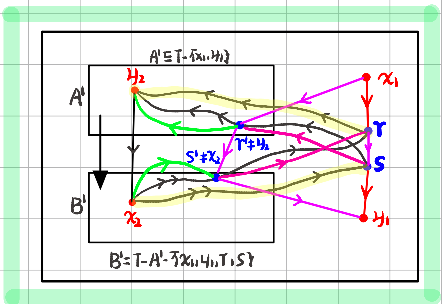


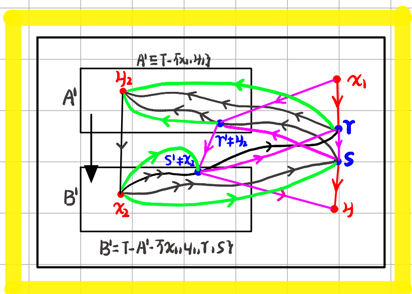
• So assume that these arcs do not exist. $\Rightarrow$ $s' \to r$, $s \to r'$

Also, we have that $x_2 \to s'$ and $r' \to y_2$. (否则, desired paths exist)







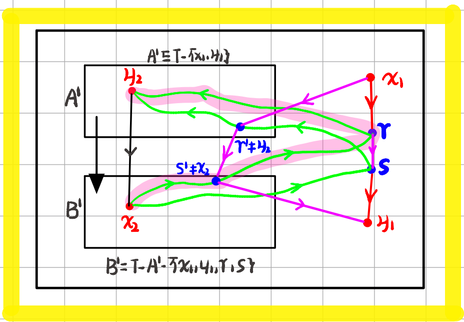


T contains the desired path if and only if

at least one of $T-\{x_2, s, r', y_2\}$, $T-\{x_2, s', r, y_2\}$ contains an (x_1, y_1)-path.

$\uparrow$

(BFS | DFS)



- $\{r', s'\}$ is also a nontrivial 2-separator of x_2, y_2 in $T-\{x_1, y_1\}$.

$\Downarrow$ similarly

desired paths exist or $x_2 \to s$ and $r \to y_2$

$\Downarrow$ in T

$x_1 \to r \to s \to y_1$, $x_1 \to r' \to s' \to y_1$; $x_2 \to s \to r' \to y_2$, $x_2 \to s' \to r \to y_2$

$\Downarrow$

- all minimal (x_1, y_1)-paths start with one of the arcs $x_1 \to r$, $x_1 \to r'$ and end with one of the arcs $s \to y_1$, $s' \to y_1$.

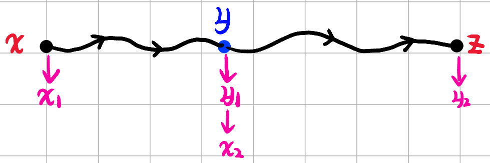
+

- $\{r, s\}$ and $\{r', s'\}$ are 2-separator of x_2, y_2.

$\square$

- Remeber that we have assume that there is no (x_2, y_2)-path in $T - \{x_1, r, s, y_1, z\}$. $(r, s$ is 2-separator of $x_2, y_2)$

Corollary 5.2. *There exists a polynomal algorithm for the following problem for semicomplete digraphs. Let T be a semicomplete digraph and let x, y, z be distinct vertices of T. Decide whether T has an (x, z)-path through y.*



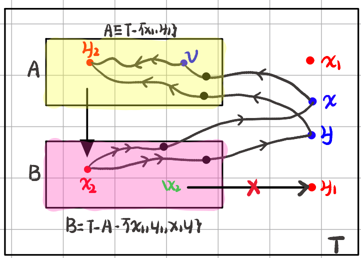
semicomplete

Lemma 4.2. *Let T be a semicomplete digraph, and let x_1, x_2, y_1, y_2 be distinct vertices such that there are two internally disjoint (x_2, y_2)-paths in $T - \{x_1, y_1\}$. Suppose that there exists a nontrivial 2-separator $\{x, y\}$ of x_2 and y_2 in $T - \{x_1, y_1\}$ such that there is no arc from $B - x_2$ to y_1, where A and B form any patition of $T - \{x_1, y_1, x, y\}$ such that $x_2 \in B$, $y_2 \in A$, and all arcs between A and B go from A to B.*

Transform T into a new semicomplete digraph T' as follows:

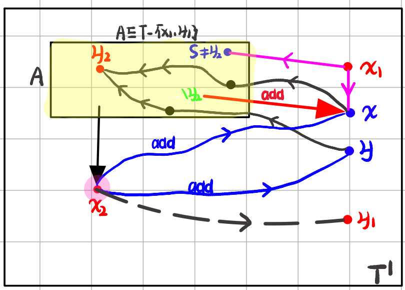
1. *If x_1 dominates some vertex in $A - y_2$ then*
   - *If there exists a vertex $b \in B - x_2$ such that $b \to x$ and there is an (x_2, y)-path in $T(B \cup \{y\} \setminus \{b\})$, then add all arcs from $A - y_2$ to x that are not present already.*
   - *If there exists a vertex $b \in B - x_2$ such that $b \to y$ and there is an (x_2, x)-path in $T(B \cup \{x\} \setminus \{b\})$, then add all arcs from $A - y_2$ to y that are not present already.*
2. *Add the arcs $x_2 \to x, x_2 \to y$ if they are not present already;*
3. *Add the arc $x_1 \to z$ for $\{z, w\} = \{x, y\}$ if $T(B \cup \{z, w, x_1\})$ has a pair of disjoint (x_1, z)–, (x_2, w)-paths, and the arc $x_1 \to z$ is not present already;*
4. *Delete the vertices of $B - x_2$.*

Call the added arcs *special* arcs. *Then the resulting semicomplete digraph T' has disjoint (x_1, y_1)-, (x_2, y_2)-paths if and only if T also has them.*



$T \to T'$

$\Rightarrow$



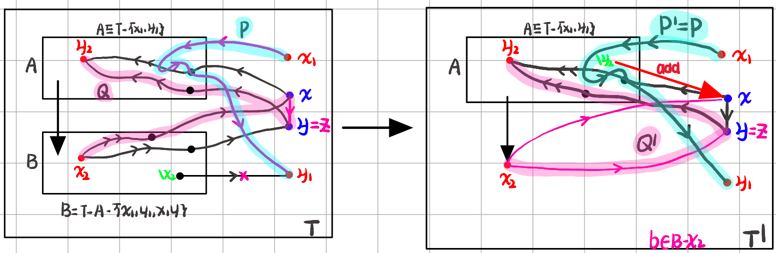
Sketch of the proof.

(T has desired paths $\Rightarrow$ T' has desired paths)

- Suppose that P and Q are disjoint (x_1,y_1)-, (x_2,y_2)-paths in T. (minimal)

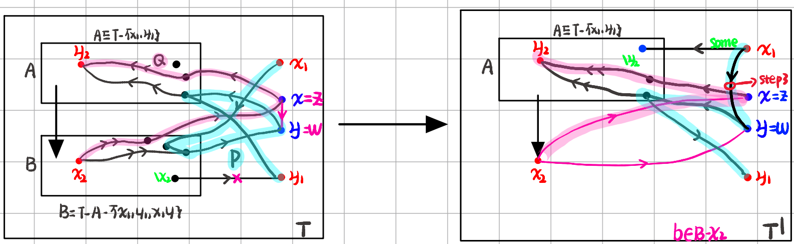
case1: If $x, y \in V(Q)$ $\Rightarrow$ then P is entirely in $T(A\cup\{x_1,y_1\})$

$\Rightarrow$ $P'=P$, $Q'=\{x_2\to z\}\cup Q[z,y_2]$ in T'



case2: Q contain only one of x,y.

- "替换路片段"将P和Q转化为T'中不相交的路P'和Q'.

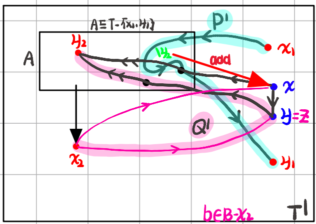


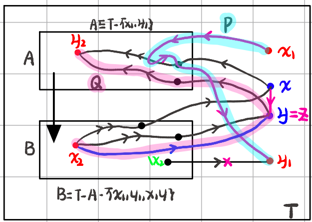
(T' has desired paths ⇒ T has desired paths)

- Suppose that P' and Q' are disjoint (x_1,y_1)-, (x_2,y_2)-paths in T'. (minimal)

Case1 P'∪Q' contains no special arcs (we added)

⇒ P=P', Q=Q'.





Case2 P'∪Q' contains special arcs (we added)

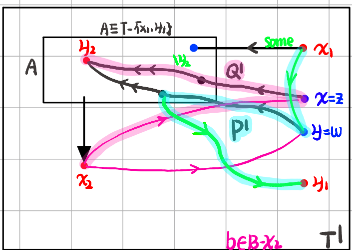
- 由Lemma 4.2的操作, operation1, operation2, operation3会产生 special arcs.

(1) P' contains the special arc obtained by operation3.

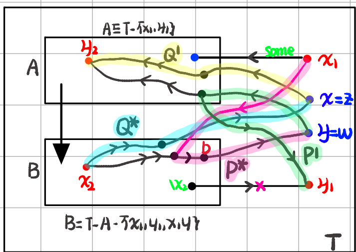
(2) P' contains the special arc obtained by operation1.

(3) P' contains no special arcs. (Q' contains)

⇒ T'→T (desired path) ✓



operate 3 ⇒



□