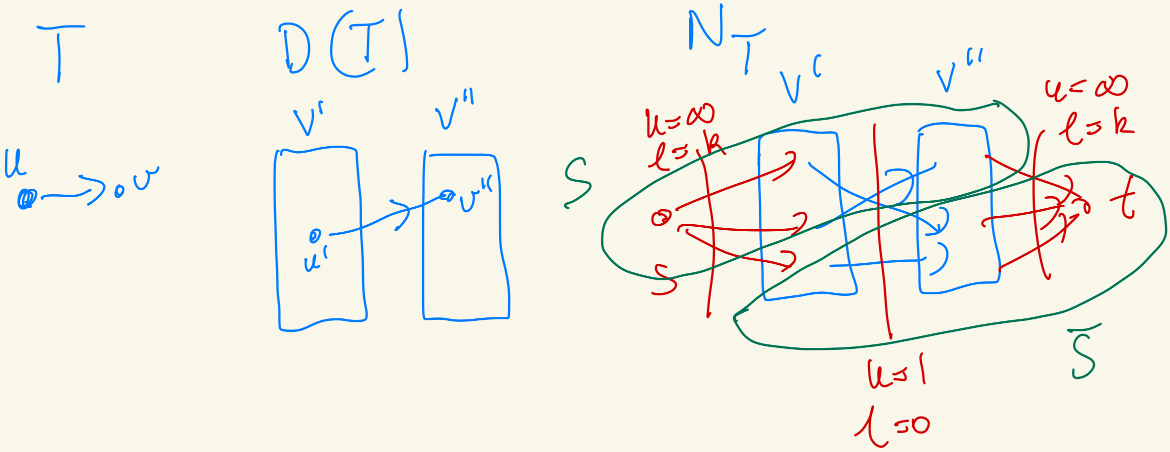
$\forall$ tournament $T = (V, A)$ s.t. $\delta^0(T) \geq k$

$\exists$ spanning subdigraph $D = (V, A')$ s.t

$\delta^0(D) \geq k$ and $|A'| \leq nk + \frac{k(k+1)}{2}$



R = min value (s,t)-flow in N_T = min $|A(D)|$ when $\delta^0(D) \geq k$

$R = \max \ell(S, \bar{S}) - u(\bar{S}, S)$ for an (s,t)-cut $(S, \bar{S})$

For a given $(S,\bar{S})$:

$X=\{v \mid v'\in\bar{S},\ v''\in S\}$ $\quad Y=\{v \mid v'\in\bar{S},\ v''\in\bar{S}\}$

$Z=\{v \mid v'\in S,\ v''\in\bar{S}\}$ $\quad W=\{v \mid v'\in S,\ v''\in S\}$

$\ell(S,\bar{S}) \le k(n+|X|-|Z|)$ and $u(\bar{S},S) \ge \binom{|X|}{2}$

$$\ell(S,\bar{S})-u(\bar{S},S) \le k(n+|X|-|Z|) - \frac{|X|(|X|-1)}{2}$$
$$= kn + \left(k|X| - \frac{|X|(|X|-1)}{2}\right) - k|Z|$$

$$\frac{d\left(k|X| - \frac{|X|(|X|-1)}{2}\right)}{d|X|} = k+\frac{1}{2}-|X|$$ max when $X=k$ or $k+1$

$X=k$:
$$\ell(S,\bar{S})-u(\bar{S},S) = kn + k\cdot k - \frac{k(k-1)}{2} - k|Z|$$
$$= kn + \frac{k(k+1)}{2} - k|Z|$$