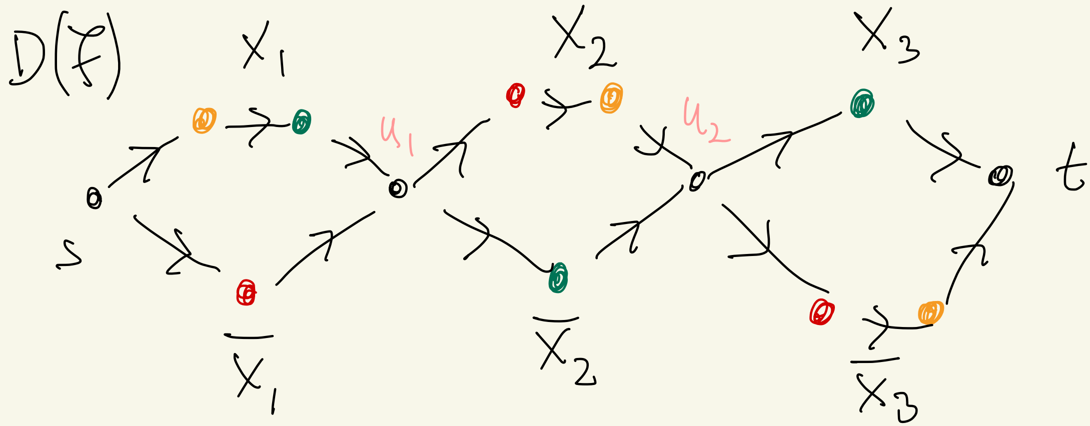
Reducing 3-SAT to a path problem

$$f = (\bar{x}_1 \vee x_2 \vee \bar{x}_3) \wedge (x_1 \vee x_2 \vee \bar{x}_3) \wedge (x_1 \vee \bar{x}_2 \vee x_3)$$



$\exists$ (s,t)-path in $D(f)$ which meets all colours

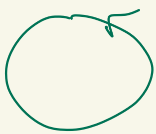
$\Updownarrow$

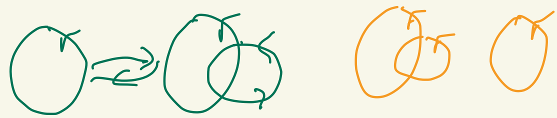
f is satisfiable

Supereulerian digraphs

A digraph $D=(V,A)$ is <u>supereulerian</u> if it contains a spanning eulerian subdigraph $D' = (V,A')$

This is equivalent to D having a spanning closed trail



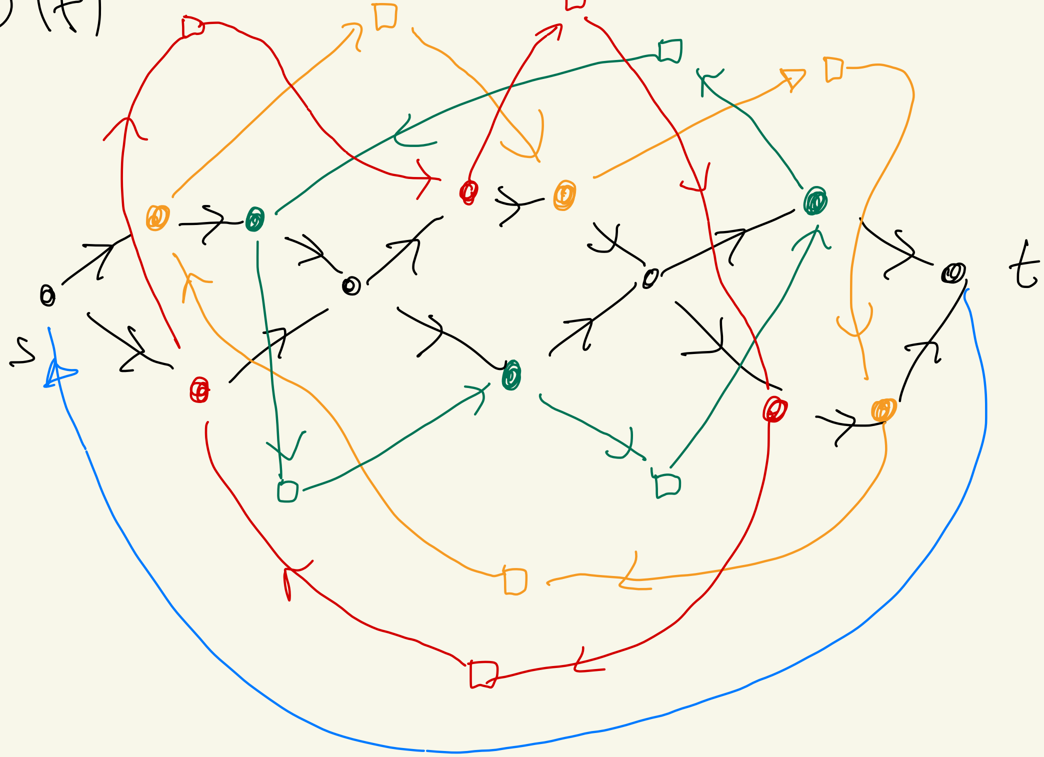


Theorem It is NP-complete to decide whether a digraph D is supereulerian

Idea: use the path construction with some extra vertices and arcs

$$f = (\bar{x}_1 \vee x_2 \vee \bar{x}_3) \wedge (x_1 \vee x_2 \vee \bar{x}_3) \wedge (x_1 \vee \bar{x}_2 \vee x_3)$$

$D^*(f)$

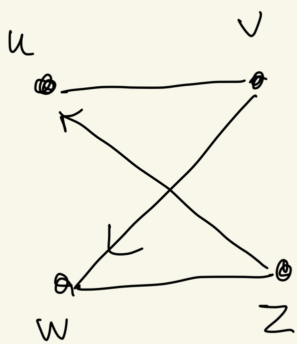


$D^*(f)$ is supereulerian

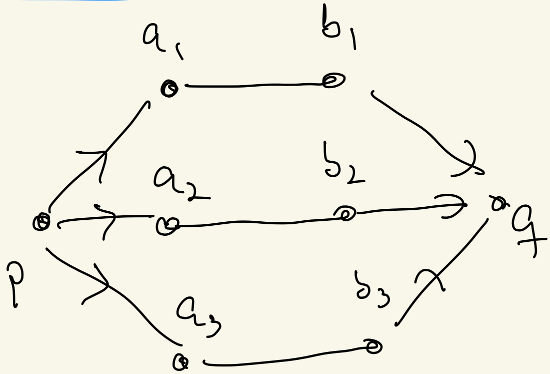
$\Updownarrow$

f is satisfiable

Completing an acyclic mixed graph M to an acyclic digraph with an (s,t)-path is NP-complete



L



W

Let D be an acyclic orientation of M

(1) if $u \to v$ in D then $z \to w$

if $w \to z$ in D then $v \to u$

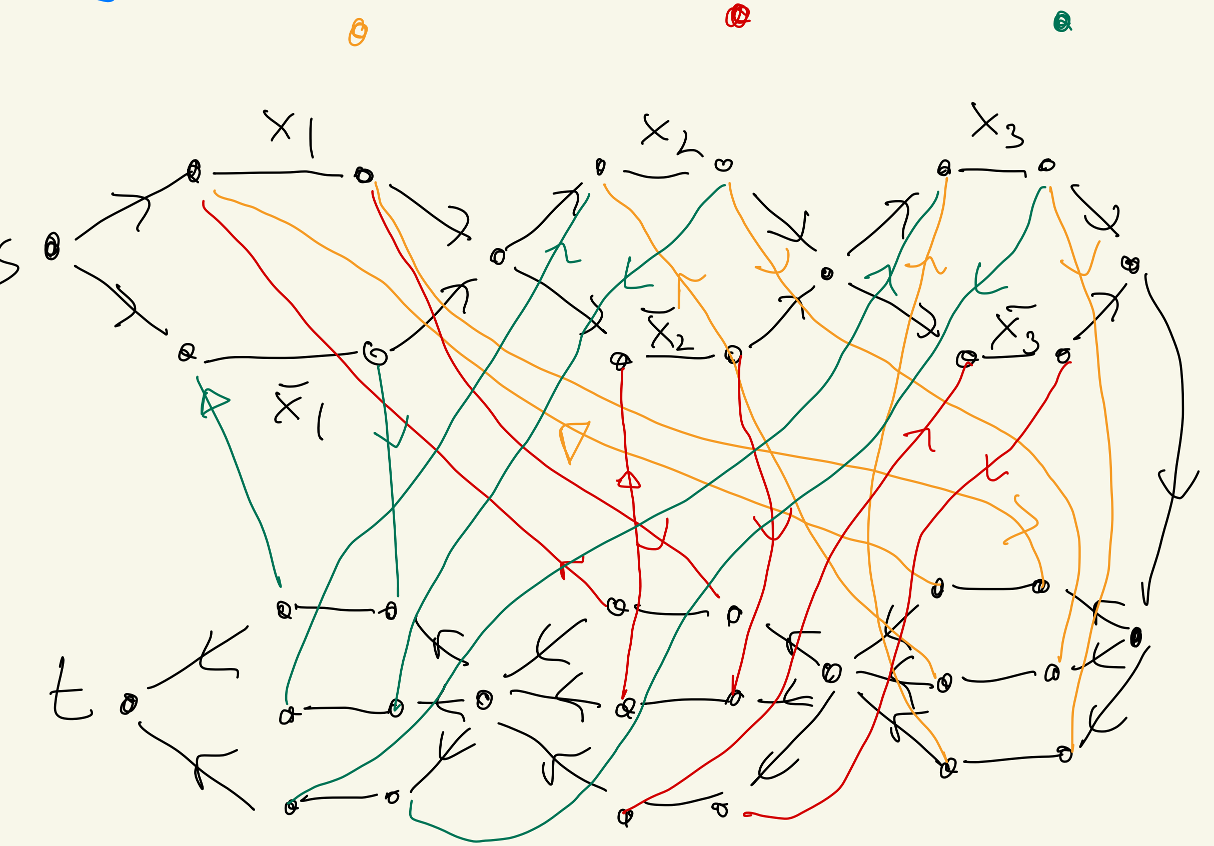
(2) if W has a (p,q)-path in D then $a_i \to b_i$ for some $i \in [3]$

<u>Idea</u> reduce 3-SAT to our problem

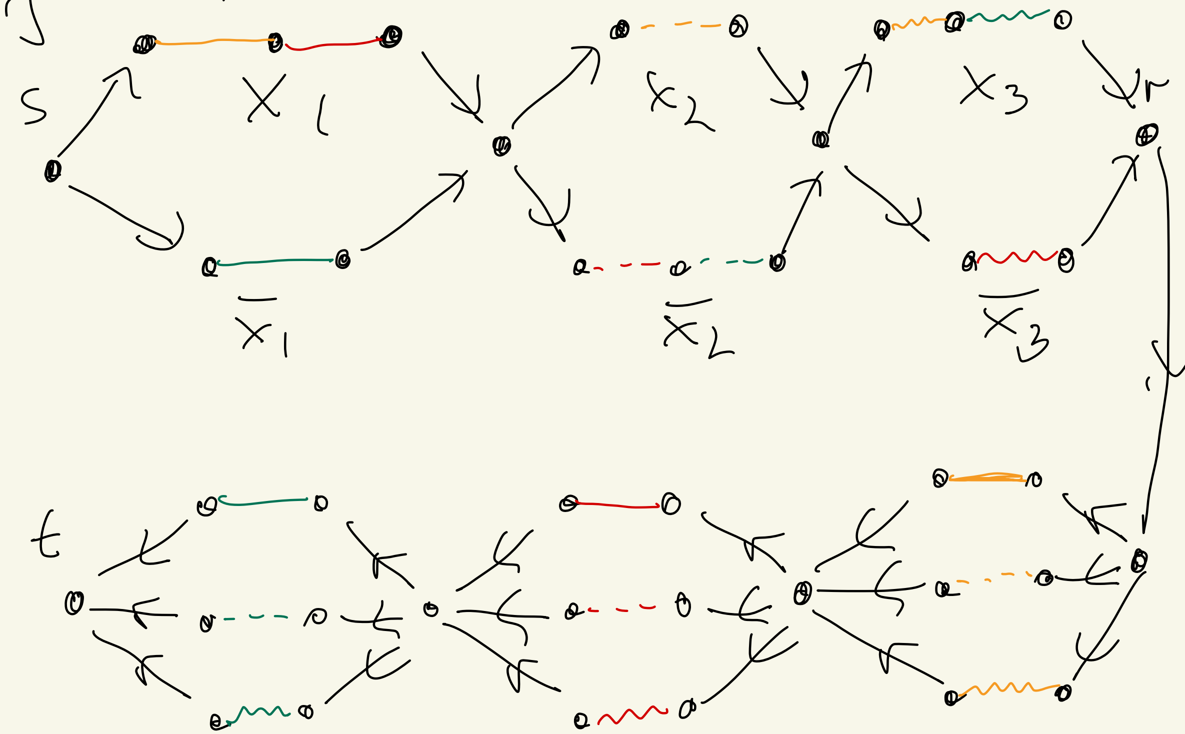
use L for literals/variable

and W for clauses

$$f = (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee x_3)$$



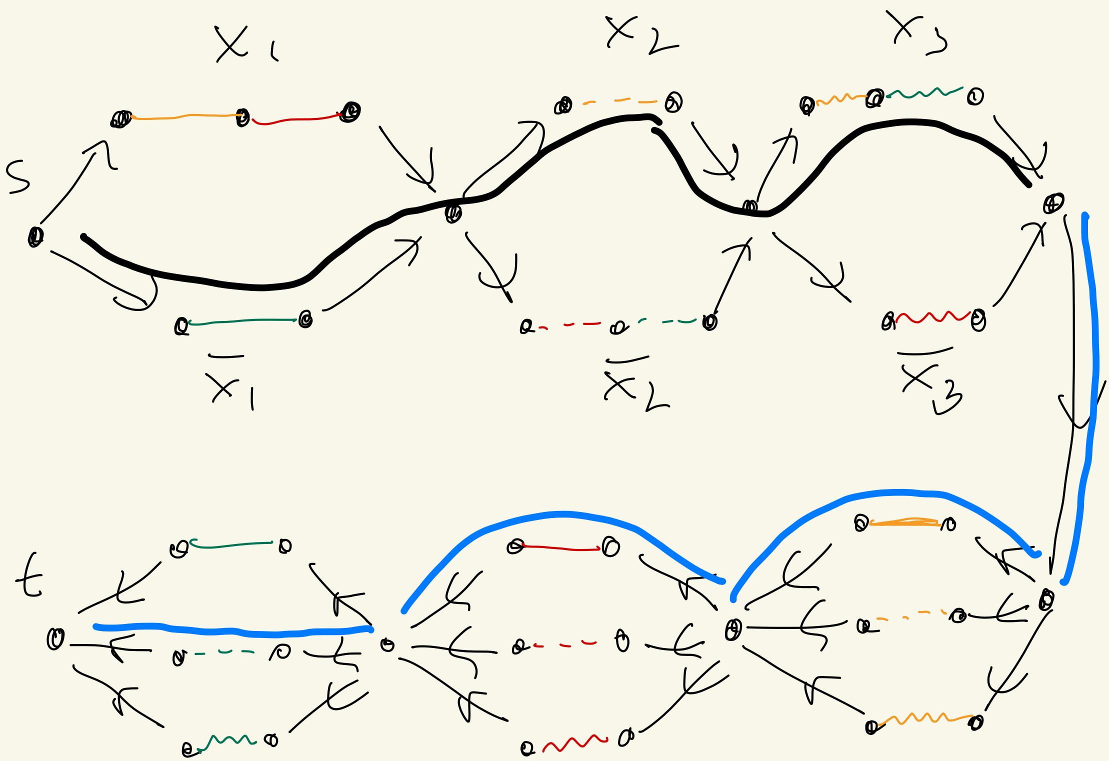
$$f = (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee x_3)$$



The desired orientation exists

$\Updownarrow$

M has an (s,r)-path which avoids at least one of the 3 coloured arcs of each clause gadget



x_1
x_2
x_3
s
$\bar{x}_1$
$\bar{x}_2$
$\bar{x}_3$
t

$x_1 \leftarrow 1$, $x_2 \leftarrow 1$, $x_3 \leftarrow 1$ satisfies

$$(x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee x_3)$$