

IKKE-EUKLIDISK GEOMETRI

OVERSIGT:

- EUKLID: DET 5. POSTULAT
- SALCHER (1667-1733)
- LAMBERT (1728-1777)
- LEGENDRE (1752-1833)
- (. SCHWEIKART (1780-1859))
- TAURINUS (1794-1874)
- GAUSS (1777-1855)
- LOBACHEVSKY (1793-1856)
- BOLYAI (1802-1860)

Forudsætninger.

Lad det være forudsat:

1. At man kan trække en ret Linie fra et hvilket som helst Punkt til et hvilket som helst Punkt.

2. At man kan forlænge en begrænset ret Linie i ret Linie ud i eet.

3. At man kan tegne en Cirkel med et hvilket som helst Centrum og en hvilken som helst Radius.

4. At alle rette Vinkler ere ligestore.

5. At, naar en ret Linie skærer to rette Linier, og de indvendige Vinkler paa samme Side ere mindre end to rette, saa mødes de to Linier, naar de forlænges ubegrænset, paa den Side, hvor de to Vinkler ligge, der ere mindre end to rette.

Almindelige Begreber.

1. Størrelser, som ere ligestore med den samme, ere indbyrdes ligestore.

2. Naar ligestore Størrelser lægges til ligestore Størrelser, ere Summerne ligestore.

PREFACE TO THE READER. *¹

* Of all who have learned mathematics, none can fail to know how great is the excellence and worth of Euclid's *Elements*. As erudite witnesses here I summon Archimedes, Apollonius, Theodosius, and others almost innumerable, writers on mathematics even to our times, who use Euclid's *Elements* as foundation long established and wholly unshaken. But this so great celebrity has not prevented many, ancients as well as moderns, and among them distinguished geometers, maintaining they had found certain blemishes in these most beautiful nor ever sufficiently praised *Elements*. Three such flecks they designate, which now I name.

The first pertains to the definition of parallels and with it the axiom which in Clavius is the thirteenth of the First Book, where Euclid says:

If a straight line falling on two straight lines, lying in the same plane, make with them two internal angles toward the same parts less than two right angles, these two straight lines infinitely produced toward those parts will meet each other.

No one doubts the truth of this proposition; but solely they accuse Euclid as to it, because he has used for it the name axiom, as if obviously from the right understanding of its terms alone came conviction. Whence not a few (withal retaining Euclid's definition of parallels) have

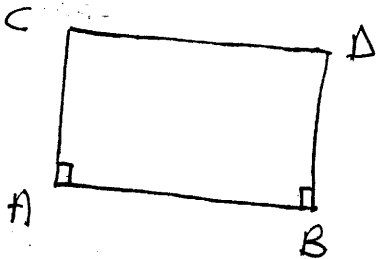
¹This symbol refers the reader to the Editor's Notes, on page 245 ff.

attempted its demonstration from those propositions of Euclid's First Book alone which precede the twenty-ninth, wherein begins the use of the controverted proposition. [x]

SACCHERI

s. 2

Viser først om den 'karakteristiske firkant':



Hvis $|AC| = |BD|$ så er
 $\angle C = \angle D$.

PROPOSITION IV.

But inversely (the figure of the preceding proposition remaining) the angles at the join CD will be right, or obtuse, or acute, according as the straight CD is equal, or less, or greater than the opposite AB.

DEFINITIONS.

Since (P. I.) the straight joining the extremities of equal perpendiculars standing upon the same straight (which we call base), makes equal [6] angles with these perpendiculars; therefore there are three hypotheses to be distinguished according to the species of these angles. And the first indeed I will call hypothesis of right angle; the second however, and the third I will call hypothesis of obtuse angle, and hypothesis of acute angle.

PROP. V, VI OG VII VISES AT HVIS $\angle C$ OG $\angle D$
ER STUMPE H#V. SPIDSE H#V. RETTE I EN FIRKANT,
SÅ VIL DE VÆRE DET I ALLE FIRKANTER

PROPOSITION IX.

*In any right-angled triangle the two acute angles remaining are, taken together, equal to one right angle, in the hypothesis of right angle; greater than one right angle, in the hypothesis of obtuse angle; but less in the hypothesis of acute angle. **

PROPOSITION XI.

Let the straight AP (as long as you choose) cut the two straights PL, AD (fig. 9), the first indeed at right angles in P, but the latter at A in any acute angle converging toward the parts PL. I say the straights AD, PL (in the hypothesis of right angle) will at length meet in some point, and indeed at a finite, or terminated distance, if they are prolonged toward those parts on which they make with the transversal AP two angles together less than two right angles.

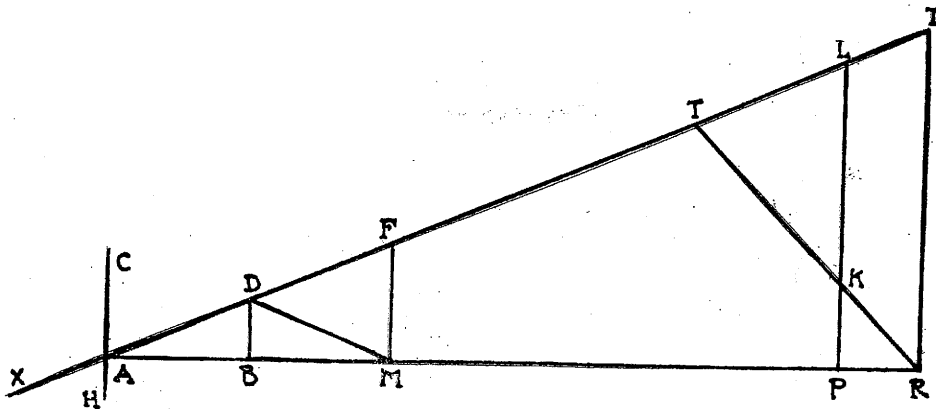


Fig. 9.

PROPOSITION XII. *

Again I say also in the hypothesis of obtuse angle the straight AD will meet the straight PL somewhere toward those parts (and indeed at a finite, or terminated distance).

PROPOSITION XIII. ✱

If the straight XA (of designated length however great) meeting two straights AD , XL , makes with them toward the same parts (fig. 11) internal angles XAD , AXL less than two right angles: I say, these two (even if neither of those angles be a right angle) at length will mutually meet in some point on the side toward those angles, and indeed at a finite, or terminated distance, if either hypothesis holds, of right angle or of obtuse angle.

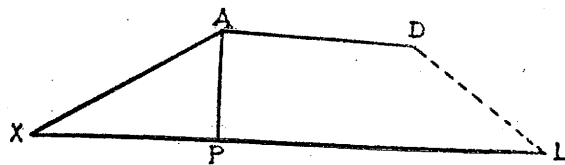


Fig. 11.

PROPOSITION XIV.

The hypothesis of obtuse angle is absolutely false, because it destroys itself.

Otherwise, and more immediately. Since from the hypothesis of obtuse angle we have proved (P. IX.) that two (fig. 11) acute angles of the triangle APX , right-angled at P , are greater than one right angle; it follows that an acute angle PAD may be assumed such, that together with the aforesaid two acute angles it makes up two right angles. But then the straight AD must (by the preceding proposition, joined to the hypothesis of obtuse angle) at length meet with this PL , or XL , regard being had to the secant, or incident AP ; which is manifestly absurd (against Eu. I. 17) if we regard the secant, or incident AX .

OM DEN 'SPIDSE VINKEL'

PROPOSITION XXXII.

Now I say there is (in the hypothesis of acute angle) a certain determinate acute angle BAX drawn under which AX (fig. 33) only at an infinite distance meets BX , and thus is a limit in part from within, in part from without; on the one hand of all those which under lesser acute angles meet the aforesaid BX at a finite distance; on the other hand also of the others

which under greater acute angles, even to a right angle inclusive, have a common perpendicular in two distinct points with BX . [69]

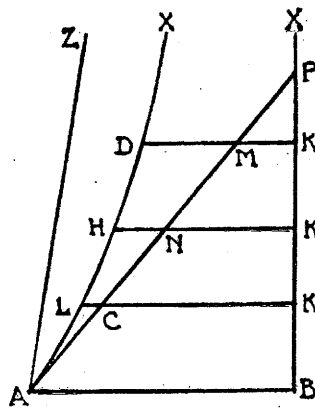


Fig. 33.

PROPOSITION XXXIII.

The hypothesis of acute angle is absolutely false; because repugnant to the nature of the straight line.

The following portion of a letter to W. BOLYAI [Dec. 17, 1799] proves that GAUSS, like SACCHERI and LAMBERT before him, had attempted to prove the truth of Postulate V. by assuming it to be false.

'As for me, I have already made some progress in my work. However the path I have chosen does not lead at all to the goal which we seek, and which you assure me you have reached.³ It seems rather to compel me to doubt the truth of geometry itself.

'It is true that I have come upon much which by most people would be held to constitute a proof: but in my eyes it proves as good as *nothing*. For example, if one could show that a rectilinear triangle is possible, whose area would be greater than any given area, then I would be ready to prove the whole of geometry absolutely rigorously.

1832:

"If I commenced by saying that I *am unable to praise this work* (by JOHANN), you would certainly be surprised for a moment. But I cannot say otherwise. To praise it, would be to praise myself. Indeed the whole contents of the work, the path taken by your son, the results to which he is led, coincide almost entirely with my meditations, which have occupied my mind partly for the last thirty or thirty-five years. So I remained quite stupefied. So far as my own work is concerned, of which up till now I have put little on paper, my intention was not to let it be published during my lifetime. Indeed the majority of people have not clear ideas upon the questions of which we are speaking, and I have found very few people who could regard with any special interest what I communicated to them on this subject. To be able to take such an interest it is first of all necessary to have devoted careful thought to the real nature of what is wanted and upon this matter almost all are most uncertain. On the other hand it was my idea to write down all this later so that at least it should not perish with me. It is therefore a pleasant surprise for me that I am spared this trouble, and I am very glad that it is just the son of my old friend, who takes the precedence of me in such a remarkable manner."