Institut for Matematik og Datalogi
Syddansk Universitet

March 22, 2019
JFB

Online Algorithms – F19 – Assignment 2

Assignment due Tuesday, April 23, 10:15

This is the second of three sets of problems (assignments) which together with the oral exam in June constitute the exam in DM860. This second set of problems must be solved individually — no working in groups.

The assignment is due at 10:15 on Tuesday, April 23. You may write this either in Danish or English. If your assignment is late, it will not be accepted. Turn in the assignment through the SDU Assignment system in Blackboard, and remember to keep your receipt.

Cheating on this assignment is viewed as cheating on an exam. Do not talk with anyone other than Joan Boyar about the assignment, and do not show your solutions to anyone else. If you have questions about the assignment, come to Joan Boyar.

Explain all of your answers.

1. Consider an artificial game, called complementary string guessing. This vaguely resembles Example 6.1 in the textbook, but is different in some respects. There are two players, the presenter and the guesser. The presenter has a binary string of length n, $\langle x_1, x_2, ..., x_n \rangle$ which is unknown to the guesser. The presenter starts and presents n. The guesser then makes a guess $y_1 \in \{0, 1\}$ for $1 - x_1$. Now it is the presenter's turn, and it presents the actual value for x_1. This continues for each bit, so in round i, the presenter reveals x_{i-1}, the guesser chooses $y_i \in \{0, 1\}$. The game ends with the presenter revealing x_n and the guesser doing nothing. The score is +1 for the guesser for each value i where $y_i = 1 - x_i$, 0 for each i where $y_i = x_i$. Thus, if the string $\langle x_1, x_2, x_3, x_4, x_5 \rangle = \langle 0, 1, 1, 0, 1 \rangle$ and $\langle y_1, y_2, y_3, y_4, y_5 \rangle = \langle 1, 1, 1, 0, 0 \rangle$, then there were two correct guesses, the first and the last, giving value 2 for the guesser.

(a) Show the game in extensive (or strategic) form for $n = 2$.

(b) What pure strategies does the guesser have for $n = 2$? List them all, along with the value for the guesser for each possible string $\langle x_1, x_2 \rangle$.

(c) One can consider this game as an online problem with the online algorithm playing the part of the guesser.

i. What is the competitive ratio of the best deterministic online algorithm for this problem? Prove this.

ii. Give a randomized algorithm with competitive ratio $1/2$ against oblivious adversaries. Prove this.

iii. Use Yao's principle to prove that no randomized algorithm can do better than than a competitive ratio of $1/2$ against oblivious adversaries.

iv. Find two deterministic algorithms are not comparable for this problem under relative worst order analysis and two that are comparable.

2. In class we considered two randomized memoryless mixed paging algorithms, both based on PERM_π. The first used the uniform distribution over all possible cyclic permutations on N pages, and the second used the uniform distribution over the two cyclic permutations $(1, 2, \ldots, N)$ and $(N, N-1, \ldots, 1)$. Both were $\frac{k+1}{2}$-competitive. Give one more distribution over the cyclic permutations on N pages which also leads to a $\frac{k+1}{2}$-competitive algorithm.

3. Is the following statement true or false?

   "If a randomized algorithm achieves competitive ratio c against an oblivious adversary, then it achieves competitive ratio c against an adaptive offline adversary."

   Justify your answer with a proof or counter-example.

4. Consider the algorithm for (unfair) dual bin packing (fixed number of bins, maximizing the number of accepted items) that behaves exactly as First-Fit would unless the item x is larger than $1/2$ and would be placed in the last bin, bin n. The algorithm *FFn* rejects such an item and is thus not fair. Show that *FFn* is better than First-Fit, according to the relative worst order ratio.

5. We know that RLRU is neither a conservative algorithm nor a marking algorithm, due to a specific sequence. Various results we have seen regarding relative worst order analysis also show this. Which ones and how?