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September 6, 2006
KSL

DM42 – Advanced Data Structures
Addition to Weekly Note 2

Defining Asymptotic Notation

Let $\mathbb{N} =$ denote the natural numbers $\{0, 1, 2, \ldots\}$ and let $\mathbb{R}^+$ the positive real numbers.

$O(f) = \{g\colon \mathbb{N} \to \mathbb{R}^+ \mid \exists c \in \mathbb{R}^+ \; \exists n_0 \in \mathbb{N} \; \forall n \in \mathbb{N}\colon \; n \geq n_0 \Rightarrow g(n) \leq cf(n)\}$

$\Omega(f) = \{g\colon \mathbb{N} \to \mathbb{R}^+ \mid \exists c \in \mathbb{R}^+ \; \exists n_0 \in \mathbb{N} \; \forall n \in \mathbb{N}\colon \; n \geq n_0 \Rightarrow g(n) \geq cf(n)\}$

$\Theta(f) = O(f) \cap \Omega(f)$

$o(f) = O(f) \setminus \Theta(f)$

$\omega(f) = \Omega(f) \setminus \Theta(f)$

$O(f(m,n)) = \{g\colon \mathbb{N}^2 \to \mathbb{R}^+ \mid \exists c \in \mathbb{R}^+ \; \exists m_0, n_0 \in \mathbb{N} \; \forall m, n \in \mathbb{N}\colon \; m \geq m_0 \wedge n \geq n_0 \Rightarrow g(m,n) \leq cf(m,n)\}$

[there are many alternative ways of defining asymptotic notation]

Repetition Problems

1. Show that $O(\log_a n) = O(\log_b n)$, where $a, b > 1$.
2. Show that $O(n) \subset O(n \log n) \subset O(n^2)$.
3. Fill in the following table with X's; and arguments.

A	B	$A \in O(B)$	$A \in o(B)$	$A \in \Omega(B)$	$A \in \omega(B)$	$A \in \Theta(B)$
$\log\log n$	$\log n$					
$(\log n)^c$	n^k					
$\frac{\log n}{\log\log n}$	$\log\log n$					
$\sqrt{n}$	$n^{\sin n}$					
$\log n!$	$\log n^n$					

   where c and k are positive constants.

4. Let c, c_1, c_2 be constants. How does $T(n)$ grow asymptotically with the following definitions of T?
   (a) $T(n) = T(\frac{n}{2}) + c$
   (b) $T(n) = 2T(\frac{n}{2}) + c$
   (c) $T(n) = 3T(\frac{n}{2}) + c$
   (d) $T(n) = T(\frac{n}{2}) + n$
   (e) $T(n) = 3T(\frac{n}{2}) + n$
   (f) $T(n) = T(n - c_1) + c_2$
   (g) $T(n) = T(n - c) + n$

   Assume that n is on some convenient form (a power of two or similar is often helpful) and that $T(1)$ is some (appropriate) constant.