

# Optimizing the Search for Repairs from Active Integrity Constraints

Luís Cruz-Filipe

Escola Superior Náutica Infante D. Henrique / CMAF / LabMAg (Portugal)

University of Southern Denmark  
September 13th, 2013

# The Problem

Databases typically pose conditions on data (“integrity constraints”). . .

# The Problem

Databases typically pose conditions on data (“integrity constraints”). . .

. . . but because of errors sometimes these conditions no longer hold.

# The Problem

Databases typically pose conditions on data (“integrity constraints”). . .

. . . but because of errors sometimes these conditions no longer hold.

## Question

How can we repair a database that no longer satisfies its integrity constraints?

# Outline

## 1 Integrity constraints

# Outline

- 1 Integrity constraints
- 2 Active integrity constraints

# Outline

- 1 Integrity constraints
- 2 Active integrity constraints
- 3 Parallellization and stratification

# Outline

- 1 Integrity constraints
- 2 Active integrity constraints
- 3 Parallellization and stratification
- 4 Conclusions

# Outline

- 1 Integrity constraints
- 2 Active integrity constraints
- 3 Parallellization and stratification
- 4 Conclusions

# A database of family relations

Consider a database with information on family relations.

# A database of family relations

Consider a database with information on family relations.

Fact

siblingOf(John, Mary)

# A database of family relations

Consider a database with information on family relations.

Fact

siblingOf(John, Mary)

This database should also contain

Missing fact

siblingOf(Mary, John)

# A database of family relations

Consider a database with information on family relations.

Fact

siblingOf(John, Mary)

This database should also contain

Missing fact

siblingOf(Mary, John)

Integrity constraint (simple)

$$\forall x \forall y. ((\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x)) \supset \perp)$$

# Can fix the problem automatically?

## Inconsistency

siblingOf(John, Mary)

$\forall x \forall y. ((\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x)) \supset \perp)$

# Can fix the problem automatically?

## Inconsistency

siblingOf(John, Mary)

$\forall x \forall y. ((\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x)) \supset \perp)$

## Solution

Add siblingOf(Mary, John)

# Can fix the problem automatically?

## Inconsistency

siblingOf(John, Mary)

$\forall x \forall y. ((\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x)) \supset \perp)$

## Solution

Add siblingOf(Mary, John)

... but is this so automatic?

# Can fix the problem automatically?

## Inconsistency

siblingOf(John, Mary)

$\forall x \forall y. ((\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x)) \supset \perp)$

## Solution

Add siblingOf(Mary, John)

... but is this so automatic?

## Another solution

Remove siblingOf(John, Mary)

# Outline

- 1 Integrity constraints
- 2 Active integrity constraints**
- 3 Parallellization and stratification
- 4 Conclusions

# Active integrity constraints

## Motivation

Specify a constraint **and** propose possible solutions.

# Active integrity constraints

## Motivation

Specify a constraint **and** propose possible solutions.

Works both ways:

# Active integrity constraints

## Motivation

Specify a constraint **and** propose possible solutions.

Works both ways:

- may express preferences

# Active integrity constraints

## Motivation

Specify a constraint **and** propose possible solutions.

Works both ways:

- may express preferences
- may eliminate options

# Family relations, revisited

## Integrity constraint

$$\forall x \forall y. ((\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x)) \supset \perp)$$

# Family relations, revisited

## Integrity constraint

$$\forall x \forall y. ((\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x)) \supset \perp)$$

## Active integrity constraint

$$\forall x \forall y. ((\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x)) \supset +\text{siblingOf}(y, x))$$

# Family relations, revisited

## Integrity constraint

$$\forall x \forall y. ((\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x)) \supset \perp)$$

## Active integrity constraint

$$\forall x \forall y. ((\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x)) \supset +\text{siblingOf}(y, x))$$

## Active integrity constraint

$$\forall x \forall y. ((\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x)) \supset -\text{siblingOf}(x, y))$$

# Family relations, revisited

## Integrity constraint

$$\forall x \forall y. ((\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x)) \supset \perp)$$

## Active integrity constraint

$$\forall x \forall y. ((\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x)) \supset \\ + \text{siblingOf}(y, x) \mid - \text{siblingOf}(x, y))$$

# Active integrity constraints

## Definition (Flesca2004)

An *Active integrity constraint* is a formula of the form

$$L_1, \dots, L_m \supset \alpha_1 \mid \dots \mid \alpha_k$$

where  $\{\alpha_1^D, \dots, \alpha_k^D\} \subseteq \{L_1, \dots, L_m\}$ .

# Active integrity constraints

## Definition (Flesca2004)

An *Active integrity constraint* is a formula of the form

$$L_1, \dots, L_m \supset \alpha_1 \mid \dots \mid \alpha_k$$

where  $\{\alpha_1^D, \dots, \alpha_k^D\} \subseteq \{L_1, \dots, L_m\}$ .

## A valid AIC

$$\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x) \supset +\text{siblingOf}(y, x)$$

# Active integrity constraints

## Definition (Flesca2004)

An *Active integrity constraint* is a formula of the form

$$L_1, \dots, L_m \supset \alpha_1 \mid \dots \mid \alpha_k$$

where  $\{\alpha_1^D, \dots, \alpha_k^D\} \subseteq \{L_1, \dots, L_m\}$ .

## A valid AIC

$$\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x) \supset +\text{siblingOf}(y, x)$$

## An invalid AIC

$$\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x) \supset -\text{siblingOf}(x, y) \mid +\text{Parent}(x)$$

# Intuitive semantics of AICs

A generic AIC

$$L_1, \dots, L_m \supset \alpha_1 \mid \dots \mid \alpha_k$$

# Intuitive semantics of AICs

## A generic AIC

$$L_1, \dots, L_m \supset \alpha_1 \mid \dots \mid \alpha_k$$

- conjunction on the left (“body”)

# Intuitive semantics of AICs

## A generic AIC

$$L_1, \dots, L_m \supset \alpha_1 \mid \dots \mid \alpha_k$$

- conjunction on the left (“body”)
- disjunction on the right (“head”)

# Intuitive semantics of AICs

## A generic AIC

$$L_1, \dots, L_m \supset \alpha_1 \mid \dots \mid \alpha_k$$

- conjunction on the left (“body”)
- disjunction on the right (“head”)
- semantics of (normal) implication

# Intuitive semantics of AICs

## A generic AIC

$$L_1, \dots, L_m \supset \alpha_1 \mid \dots \mid \alpha_k$$

- conjunction on the left (“body”)
- disjunction on the right (“head”)
- semantics of (normal) implication
- holds iff one of the  $L_i$ s fails (but...)

# Intuitive semantics of AICs

## A generic AIC

$$L_1, \dots, L_m \supset \alpha_1 \mid \dots \mid \alpha_k$$

- conjunction on the left (“body”)
- disjunction on the right (“head”)
- semantics of (normal) implication
- holds iff one of the  $L_i$ s fails (but...)
- $\{\alpha_1^D, \dots, \alpha_k^D\}$  are *updatable* literals

# Repairs

## Definition (Caroprese et al., 2006)

Let  $\mathcal{I}$  be a database and  $\eta$  be a set of (A)ICs. A *weak repair* for  $\mathcal{I}$  and  $\eta$  is a consistent set  $\mathcal{U}$  of update actions such that:

# Repairs

## Definition (Caroprese et al., 2006)

Let  $\mathcal{I}$  be a database and  $\eta$  be a set of (A)ICs. A *weak repair* for  $\mathcal{I}$  and  $\eta$  is a consistent set  $\mathcal{U}$  of update actions such that:

- $\mathcal{U}$  consists of essential actions only

# Repairs

## Definition (Caroprese et al., 2006)

Let  $\mathcal{I}$  be a database and  $\eta$  be a set of (A)ICs. A *weak repair* for  $\mathcal{I}$  and  $\eta$  is a consistent set  $\mathcal{U}$  of update actions such that:

- $\mathcal{U}$  consists of essential actions only
- $\mathcal{I} \circ \mathcal{U} \models \eta$

# Repairs

## Definition (Caroprese et al., 2006)

Let  $\mathcal{I}$  be a database and  $\eta$  be a set of (A)ICs. A *weak repair* for  $\mathcal{I}$  and  $\eta$  is a consistent set  $\mathcal{U}$  of update actions such that:

- $\mathcal{U}$  consists of essential actions only
- $\mathcal{I} \circ \mathcal{U} \models \eta$

(Beware of the notation.)

# Repairs

## Definition (Caroprese et al., 2006)

Let  $\mathcal{I}$  be a database and  $\eta$  be a set of (A)ICs. A *weak repair* for  $\mathcal{I}$  and  $\eta$  is a consistent set  $\mathcal{U}$  of update actions such that:

- $\mathcal{U}$  consists of essential actions only
- $\mathcal{I} \circ \mathcal{U} \models \eta$

(Beware of the notation.)

## Definition

A *repair* is a weak repair that is minimal w.r.t. inclusion.

# Family relations, yet again

## Inconsistency

siblingOf(John, Mary)

$\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x) \supset +\text{siblingOf}(y, x)$

# Family relations, yet again

## Inconsistency

siblingOf(John, Mary)

$\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x) \supset +\text{siblingOf}(y, x)$

## A repair

$+\text{siblingOf}(\text{Mary}, \text{John})$

# Family relations, yet again

## Inconsistency

siblingOf(John, Mary)

$\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x) \supset +\text{siblingOf}(y, x)$

## Another repair

$\neg \text{siblingOf}(\text{John}, \text{Mary})$

# Family relations, yet again

## Inconsistency

siblingOf(John, Mary)

$\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x) \supset +\text{siblingOf}(y, x)$

## A weak repair

$+\text{siblingOf}(\text{Mary}, \text{John}), +\text{Parent}(\text{John})$

# Family relations, yet again

## Inconsistency

$\text{siblingOf}(\text{John}, \text{Mary})$

$\text{siblingOf}(x, y) \wedge \neg \text{siblingOf}(y, x) \supset +\text{siblingOf}(y, x)$

## Not a weak repair

$+\text{siblingOf}(\text{Mary}, \text{John}), -\text{siblingOf}(\text{John}, \text{Mary})$

# Finding repairs

## Algorithm

- 1 Choose a set  $\mathcal{U}$  of update actions (based on  $\mathcal{I}$ )
- 2 Compute  $\mathcal{I} \circ \mathcal{U}$
- 3 Check if all AICs in  $\eta$  hold

# Finding repairs

## Algorithm

- 1 Choose a set  $\mathcal{U}$  of update actions (based on  $\mathcal{I}$ )
- 2 Compute  $\mathcal{I} \circ \mathcal{U}$
- 3 Check if all AICs in  $\eta$  hold

Each step can be done in polynomial time on  $\mathcal{I}$  and  $\eta$ .

# Finding repairs

## Algorithm

- 1 Choose a set  $\mathcal{U}$  of update actions (based on  $\mathcal{I}$ )
- 2 Compute  $\mathcal{I} \circ \mathcal{U}$
- 3 Check if all AICs in  $\eta$  hold

Each step can be done in polynomial time on  $\mathcal{I}$  and  $\eta$ .

Finding weak repairs is NP-complete.

# Declarative semantics

The notion of repair ignores the head of the AIC.

# Declarative semantics

The notion of repair ignores the head of the AIC.

Caroprese et al., 2006/2011

- *founded* repairs take into account the actions in the head

# Declarative semantics

The notion of repair ignores the head of the AIC.

Caroprese et al., 2006/2011

- *founded* repairs take into account the actions in the head
- *justified* repairs avoid justification circles

# Founded repairs (I)

Intuitively: if  $\mathcal{U}$  is founded, then removing an action from  $\mathcal{U}$  causes some AIC with that action in the head to be violated.

# Founded repairs (I)

Intuitively: if  $\mathcal{U}$  is founded, then removing an action from  $\mathcal{U}$  causes some AIC with that action in the head to be violated.

## Definition

A set of update actions  $\mathcal{U}$  is founded w.r.t.  $\mathcal{I}$  and  $\eta$  if, for every  $\alpha \in \mathcal{U}$ , there is a rule  $r \in \eta$  such that  $\alpha \in \text{head}(r)$  and

$$\mathcal{I} \circ (\mathcal{U} \setminus \{\alpha\}) \not\models r.$$

# Founded repairs (I)

Intuitively: if  $\mathcal{U}$  is founded, then removing an action from  $\mathcal{U}$  causes some AIC with that action in the head to be violated.

## Definition

A set of update actions  $\mathcal{U}$  is founded w.r.t.  $\mathcal{I}$  and  $\eta$  if, for every  $\alpha \in \mathcal{U}$ , there is a rule  $r \in \eta$  such that  $\alpha \in \text{head}(r)$  and

$$\mathcal{I} \circ (\mathcal{U} \setminus \{\alpha\}) \not\models r.$$

(Equivalent to the original definition.)

# Founded repairs (I)

Intuitively: if  $\mathcal{U}$  is founded, then removing an action from  $\mathcal{U}$  causes some AIC with that action in the head to be violated.

## Definition

A set of update actions  $\mathcal{U}$  is founded w.r.t.  $\mathcal{I}$  and  $\eta$  if, for every  $\alpha \in \mathcal{U}$ , there is a rule  $r \in \eta$  such that  $\alpha \in \text{head}(r)$  and

$$\mathcal{I} \circ (\mathcal{U} \setminus \{\alpha\}) \not\models r.$$

(Equivalent to the original definition.)

## Definition

A (weak) repair that is founded is called a founded (weak) repair.

# Founded repairs (II)

## Example

Take  $\mathcal{I} = \{a, b\}$  and

$$r_1 : a, \text{not } b \supset -a$$

$$r_2 : \text{not } a, b \supset -b$$

$$r_3 : a, \text{not } c \supset +c$$

$$r_4 : b, \text{not } c \supset +c$$

## Founded repairs (II)

### Example

Take  $\mathcal{I} = \{a, b\}$  and

$$r_1 : a, \text{not } b \supset -a$$

$$r_2 : \text{not } a, b \supset -b$$

$$r_3 : a, \text{not } c \supset +c$$

$$r_4 : b, \text{not } c \supset +c$$

Then  $\{+c\}$  is a founded repair,

## Founded repairs (II)

### Example

Take  $\mathcal{I} = \{a, b\}$  and

$$r_1 : a, \text{not } b \supset -a$$

$$r_2 : \text{not } a, b \supset -b$$

$$r_3 : a, \text{not } c \supset +c$$

$$r_4 : b, \text{not } c \supset +c$$

Then  $\{+c\}$  is a founded repair, but so is  $\{-a, -b\}$ .

## Founded repairs (II)

### Example

Take  $\mathcal{I} = \{a, b\}$  and

$$r_1 : a, \text{not } b \supset -a$$

$$r_3 : a, \text{not } c \supset +c$$

$$r_2 : \text{not } a, b \supset -b$$

$$r_4 : b, \text{not } c \supset +c$$

Then  $\{+c\}$  is a founded repair, but so is  $\{-a, -b\}$ .

In  $\{-a, -b\}$  we have a *circularity of support*.

## Founded repairs (II)

### Example

Take  $\mathcal{I} = \{a, b\}$  and

$$r_1 : a, \text{not } b \supset -a$$

$$r_3 : a, \text{not } c \supset +c$$

$$r_2 : \text{not } a, b \supset -b$$

$$r_4 : b, \text{not } c \supset +c$$

Then  $\{+c\}$  is a founded repair, but so is  $\{-a, -b\}$ .

In  $\{-a, -b\}$  we have a *circularity of support*.

It is a founded repair that is not justified.

# Justified repairs

The definition is meant to avoid circularity of support.

# Justified repairs

The definition is meant to avoid circularity of support.

It is not intuitive and there is no motivation in the references.

# Justified repairs

The definition is meant to avoid circularity of support.

It is not intuitive and there is no motivation in the references.

## Too restrictive?

Take  $\mathcal{I} = \{a, b\}$  and

$$r_1 : a, b \supset -a \quad r_2 : a, \text{not } b \supset -a \quad r_3 : \text{not } a, b \supset -b$$

# Justified repairs

The definition is meant to avoid circularity of support.

It is not intuitive and there is no motivation in the references.

## Too restrictive?

Take  $\mathcal{I} = \{a, b\}$  and

$$r_1 : a, b \supset -a \quad r_2 : a, \text{not } b \supset -a \quad r_3 : \text{not } a, b \supset -b$$

Again  $\{-a, -b\}$  is a founded repair that is not justified.

# Complexity

Deciding whether there is a founded weak repair for  $\mathcal{I}$  and  $\eta$  is NP-complete.

# Complexity

Deciding whether there is a founded weak repair for  $\mathcal{I}$  and  $\eta$  is NP-complete.

Deciding whether there is a founded repair for  $\mathcal{I}$  and  $\eta$  is  $\Sigma_P^2$ -complete.

# Complexity

Deciding whether there is a founded weak repair for  $\mathcal{I}$  and  $\eta$  is NP-complete.

Deciding whether there is a founded repair for  $\mathcal{I}$  and  $\eta$  is  $\Sigma_P^2$ -complete.

Deciding whether there is a justified (weak) repair for  $\mathcal{I}$  and  $\eta$  is  $\Sigma_P^2$ -complete.

# Outline

- 1 Integrity constraints
- 2 Active integrity constraints
- 3 Parallellization and stratification**
- 4 Conclusions

# Motivation

Reduce the size of the problem by splitting the set of AICs into smaller sets.

# Motivation

Reduce the size of the problem by splitting the set of AICs into smaller sets.

Goals:

# Motivation

Reduce the size of the problem by splitting the set of AICs into smaller sets.

Goals:

- do not lose repairs;

# Motivation

Reduce the size of the problem by splitting the set of AICs into smaller sets.

Goals:

- do not lose repairs;
- efficient combination of results.

# Independence

## Definition

Two AICs  $r_1$  and  $r_2$  are *independent*,  $r_1 \perp\!\!\!\perp r_2$ , if the literals in their bodies do not share atoms.

# Independence

## Definition

Two AICs  $r_1$  and  $r_2$  are *independent*,  $r_1 \perp\!\!\!\perp r_2$ , if the literals in their bodies do not share atoms.

Two sets of AICs  $\eta_1$  and  $\eta_2$  are *independent*,  $\eta_1 \perp\!\!\!\perp \eta_2$ , if  $r_1 \perp\!\!\!\perp r_2$  for every  $r_1 \in \eta_1$  and  $r_2 \in \eta_2$ .

# Independence

## Definition

Two AICs  $r_1$  and  $r_2$  are *independent*,  $r_1 \perp\!\!\!\perp r_2$ , if the literals in their bodies do not share atoms.

Two sets of AICs  $\eta_1$  and  $\eta_2$  are *independent*,  $\eta_1 \perp\!\!\!\perp \eta_2$ , if  $r_1 \perp\!\!\!\perp r_2$  for every  $r_1 \in \eta_1$  and  $r_2 \in \eta_2$ .

- No attention to the heads of the rules.

# Independence

## Definition

Two AICs  $r_1$  and  $r_2$  are *independent*,  $r_1 \perp\!\!\!\perp r_2$ , if the literals in their bodies do not share atoms.

Two sets of AICs  $\eta_1$  and  $\eta_2$  are *independent*,  $\eta_1 \perp\!\!\!\perp \eta_2$ , if  $r_1 \perp\!\!\!\perp r_2$  for every  $r_1 \in \eta_1$  and  $r_2 \in \eta_2$ .

- No attention to the heads of the rules.
- Not affected by the database.

# Independence vs. parallelization (I)

## Theorem

*Let  $\eta = \eta_1 \cup \eta_2$  with  $\eta_1 \perp\!\!\!\perp \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}$  be a weak repair for  $\mathcal{I}$  and  $\eta$ .*

# Independence vs. parallellization (I)

## Theorem

*Let  $\eta = \eta_1 \cup \eta_2$  with  $\eta_1 \perp\!\!\!\perp \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}$  be a weak repair for  $\mathcal{I}$  and  $\eta$ .*

*Define  $\mathcal{U}_i$  as the set of actions in  $\mathcal{U}$  affecting literals in the bodies of rules in  $\eta_i$ , for  $i = 1, 2$ .*

# Independence vs. parallellization (I)

## Theorem

*Let  $\eta = \eta_1 \cup \eta_2$  with  $\eta_1 \perp\!\!\!\perp \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}$  be a weak repair for  $\mathcal{I}$  and  $\eta$ .*

*Define  $\mathcal{U}_i$  as the set of actions in  $\mathcal{U}$  affecting literals in the bodies of rules in  $\eta_i$ , for  $i = 1, 2$ .*

*Then:*

# Independence vs. parallellization (I)

## Theorem

Let  $\eta = \eta_1 \cup \eta_2$  with  $\eta_1 \perp\!\!\!\perp \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}$  be a weak repair for  $\mathcal{I}$  and  $\eta$ .

Define  $\mathcal{U}_i$  as the set of actions in  $\mathcal{U}$  affecting literals in the bodies of rules in  $\eta_i$ , for  $i = 1, 2$ .

Then:

- each  $\mathcal{U}_i$  is a weak repair for  $\mathcal{I}$  and  $\eta_i$ ;

# Independence vs. parallellization (I)

## Theorem

Let  $\eta = \eta_1 \cup \eta_2$  with  $\eta_1 \perp\!\!\!\perp \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}$  be a weak repair for  $\mathcal{I}$  and  $\eta$ .

Define  $\mathcal{U}_i$  as the set of actions in  $\mathcal{U}$  affecting literals in the bodies of rules in  $\eta_i$ , for  $i = 1, 2$ .

Then:

- each  $\mathcal{U}_i$  is a weak repair for  $\mathcal{I}$  and  $\eta_i$ ;
- if  $\mathcal{U}$  is a repair, then so is each  $\mathcal{U}_i$ ;

# Independence vs. parallellization (I)

## Theorem

Let  $\eta = \eta_1 \cup \eta_2$  with  $\eta_1 \perp\!\!\!\perp \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}$  be a weak repair for  $\mathcal{I}$  and  $\eta$ .

Define  $\mathcal{U}_i$  as the set of actions in  $\mathcal{U}$  affecting literals in the bodies of rules in  $\eta_i$ , for  $i = 1, 2$ .

Then:

- each  $\mathcal{U}_i$  is a weak repair for  $\mathcal{I}$  and  $\eta_i$ ;
- if  $\mathcal{U}$  is a repair, then so is each  $\mathcal{U}_i$ ;
- if  $\mathcal{U}$  is founded, then so is each  $\mathcal{U}_i$ ;

# Independence vs. parallelization (I)

## Theorem

Let  $\eta = \eta_1 \cup \eta_2$  with  $\eta_1 \perp\!\!\!\perp \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}$  be a weak repair for  $\mathcal{I}$  and  $\eta$ .

Define  $\mathcal{U}_i$  as the set of actions in  $\mathcal{U}$  affecting literals in the bodies of rules in  $\eta_i$ , for  $i = 1, 2$ .

Then:

- each  $\mathcal{U}_i$  is a weak repair for  $\mathcal{I}$  and  $\eta_i$ ;
- if  $\mathcal{U}$  is a repair, then so is each  $\mathcal{U}_i$ ;
- if  $\mathcal{U}$  is founded, then so is each  $\mathcal{U}_i$ ;
- if  $\mathcal{U}$  is justified, then so is each  $\mathcal{U}_i$ .

# Independence vs. parallellization (I)

## Theorem

Let  $\eta = \eta_1 \cup \eta_2$  with  $\eta_1 \perp\!\!\!\perp \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}$  be a weak repair for  $\mathcal{I}$  and  $\eta$ .

Define  $\mathcal{U}_i$  as the set of actions in  $\mathcal{U}$  affecting literals in the bodies of rules in  $\eta_i$ , for  $i = 1, 2$ .

Then:

- each  $\mathcal{U}_i$  is a weak repair for  $\mathcal{I}$  and  $\eta_i$ ;
- if  $\mathcal{U}$  is a repair, then so is each  $\mathcal{U}_i$ ;
- if  $\mathcal{U}$  is founded, then so is each  $\mathcal{U}_i$ ;
- if  $\mathcal{U}$  is justified, then so is each  $\mathcal{U}_i$ .

Furthermore, if every action in  $\mathcal{U}$  affects a literal in the body of a rule in  $\eta$ , then  $\mathcal{U} = \mathcal{U}_1 \cup \mathcal{U}_2$ .

# Independence vs. parallelization (I)

## Theorem

Let  $\eta = \eta_1 \cup \eta_2$  with  $\eta_1 \perp\!\!\!\perp \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}$  be a weak repair for  $\mathcal{I}$  and  $\eta$ .

Define  $\mathcal{U}_i$  as the set of actions in  $\mathcal{U}$  affecting literals in the bodies of rules in  $\eta_i$ , for  $i = 1, 2$ .

Then:

- each  $\mathcal{U}_i$  is a weak repair for  $\mathcal{I}$  and  $\eta_i$ ;
- if  $\mathcal{U}$  is a repair, then so is each  $\mathcal{U}_i$ ;
- if  $\mathcal{U}$  is founded, then so is each  $\mathcal{U}_i$ ;
- if  $\mathcal{U}$  is justified, then so is each  $\mathcal{U}_i$ .

Furthermore, if every action in  $\mathcal{U}$  affects a literal in the body of a rule in  $\eta$ , then  $\mathcal{U} = \mathcal{U}_1 \cup \mathcal{U}_2$ . This hypothesis is (very) reasonable in practice.

## Independence vs. parallelization (II)

### Proof (sketch).

Given  $r_1 \in \eta_1$  and  $r_2 \in \eta_2$ , changing the logical values of literals in the body of  $r_1$  cannot affect the semantics of  $r_2$  and vice-versa.

## Independence vs. parallelization (II)

### Proof (sketch).

Given  $r_1 \in \eta_1$  and  $r_2 \in \eta_2$ , changing the logical values of literals in the body of  $r_1$  cannot affect the semantics of  $r_2$  and vice-versa. This implies that  $\mathcal{U}_i$  is a weak repair for  $\mathcal{I}$  and  $\eta_i$ .

## Independence vs. parallellization (II)

### Proof (sketch).

Given  $r_1 \in \eta_1$  and  $r_2 \in \eta_2$ , changing the logical values of literals in the body of  $r_1$  cannot affect the semantics of  $r_2$  and vice-versa.

This implies that  $\mathcal{U}_i$  is a weak repair for  $\mathcal{I}$  and  $\eta_i$ .

This also implies that, if  $\mathcal{U}$  is minimal, then so must  $\mathcal{U}_1$  and  $\mathcal{U}_2$  be.

## Independence vs. parallellization (II)

### Proof (sketch).

Given  $r_1 \in \eta_1$  and  $r_2 \in \eta_2$ , changing the logical values of literals in the body of  $r_1$  cannot affect the semantics of  $r_2$  and vice-versa.

This implies that  $\mathcal{U}_i$  is a weak repair for  $\mathcal{I}$  and  $\eta_i$ .

This also implies that, if  $\mathcal{U}$  is minimal, then so must  $\mathcal{U}_1$  and  $\mathcal{U}_2$  be.

For foundedness, take  $\alpha \in \mathcal{U}_1$ .

## Independence vs. parallellization (II)

### Proof (sketch).

Given  $r_1 \in \eta_1$  and  $r_2 \in \eta_2$ , changing the logical values of literals in the body of  $r_1$  cannot affect the semantics of  $r_2$  and vice-versa.

This implies that  $\mathcal{U}_i$  is a weak repair for  $\mathcal{I}$  and  $\eta_i$ .

This also implies that, if  $\mathcal{U}$  is minimal, then so must  $\mathcal{U}_1$  and  $\mathcal{U}_2$  be.

For foundedness, take  $\alpha \in \mathcal{U}_1$ . Since  $\mathcal{U}$  is founded, there is a rule  $r \in \eta$  such that  $\mathcal{I} \circ (\mathcal{U} \setminus \{\alpha\}) \not\models r$ .

## Independence vs. parallellization (II)

### Proof (sketch).

Given  $r_1 \in \eta_1$  and  $r_2 \in \eta_2$ , changing the logical values of literals in the body of  $r_1$  cannot affect the semantics of  $r_2$  and vice-versa.

This implies that  $\mathcal{U}_i$  is a weak repair for  $\mathcal{I}$  and  $\eta_i$ .

This also implies that, if  $\mathcal{U}$  is minimal, then so must  $\mathcal{U}_1$  and  $\mathcal{U}_2$  be.

For foundedness, take  $\alpha \in \mathcal{U}_1$ . Since  $\mathcal{U}$  is founded, there is a rule  $r \in \eta$  such that  $\mathcal{I} \circ (\mathcal{U} \setminus \{\alpha\}) \not\models r$ . Necessarily  $r \in \eta_1$

## Independence vs. parallellization (II)

### Proof (sketch).

Given  $r_1 \in \eta_1$  and  $r_2 \in \eta_2$ , changing the logical values of literals in the body of  $r_1$  cannot affect the semantics of  $r_2$  and vice-versa.

This implies that  $\mathcal{U}_i$  is a weak repair for  $\mathcal{I}$  and  $\eta_i$ .

This also implies that, if  $\mathcal{U}$  is minimal, then so must  $\mathcal{U}_1$  and  $\mathcal{U}_2$  be.

For foundedness, take  $\alpha \in \mathcal{U}_1$ . Since  $\mathcal{U}$  is founded, there is a rule  $r \in \eta$  such that  $\mathcal{I} \circ (\mathcal{U} \setminus \{\alpha\}) \not\models r$ . Necessarily  $r \in \eta_1$  and  $\mathcal{I} \circ (\mathcal{U}_1 \setminus \{\alpha\}) \not\models r$ .

## Independence vs. parallelization (II)

### Proof (sketch).

Given  $r_1 \in \eta_1$  and  $r_2 \in \eta_2$ , changing the logical values of literals in the body of  $r_1$  cannot affect the semantics of  $r_2$  and vice-versa.

This implies that  $\mathcal{U}_i$  is a weak repair for  $\mathcal{I}$  and  $\eta_i$ .

This also implies that, if  $\mathcal{U}$  is minimal, then so must  $\mathcal{U}_1$  and  $\mathcal{U}_2$  be.

For foundedness, take  $\alpha \in \mathcal{U}_1$ . Since  $\mathcal{U}$  is founded, there is a rule  $r \in \eta$  such that  $\mathcal{I} \circ (\mathcal{U} \setminus \{\alpha\}) \not\models r$ . Necessarily  $r \in \eta_1$  and  $\mathcal{I} \circ (\mathcal{U}_1 \setminus \{\alpha\}) \not\models r$ . Therefore  $\mathcal{U}_1$  is founded.

## Independence vs. parallellization (II)

### Proof (sketch).

Given  $r_1 \in \eta_1$  and  $r_2 \in \eta_2$ , changing the logical values of literals in the body of  $r_1$  cannot affect the semantics of  $r_2$  and vice-versa.

This implies that  $\mathcal{U}_i$  is a weak repair for  $\mathcal{I}$  and  $\eta_i$ .

This also implies that, if  $\mathcal{U}$  is minimal, then so must  $\mathcal{U}_1$  and  $\mathcal{U}_2$  be.

For foundedness, take  $\alpha \in \mathcal{U}_1$ . Since  $\mathcal{U}$  is founded, there is a rule  $r \in \eta$  such that  $\mathcal{I} \circ (\mathcal{U} \setminus \{\alpha\}) \not\models r$ . Necessarily  $r \in \eta_1$  and  $\mathcal{I} \circ (\mathcal{U}_1 \setminus \{\alpha\}) \not\models r$ . Therefore  $\mathcal{U}_1$  is founded.

This means that we can parallellize the search for repairs without losing solutions.

## Independence vs. parallelization (III)

### Theorem

*Let  $\eta = \eta_1 \cup \eta_2$  with  $\eta_1 \perp\!\!\!\perp \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}_i$  be weak repairs for  $\mathcal{I}$  and  $\eta_i$ , for  $i = 1, 2$ , such that all actions in  $\mathcal{U}_i$  affect a literal in the body of a rule in  $\eta_i$ .*

## Independence vs. parallelization (III)

### Theorem

*Let  $\eta = \eta_1 \cup \eta_2$  with  $\eta_1 \perp\!\!\!\perp \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}_i$  be weak repairs for  $\mathcal{I}$  and  $\eta_i$ , for  $i = 1, 2$ , such that all actions in  $\mathcal{U}_i$  affect a literal in the body of a rule in  $\eta_i$ .*

*Define  $\mathcal{U} = \mathcal{U}_1 \cup \mathcal{U}_2$ .*

## Independence vs. parallelization (III)

### Theorem

Let  $\eta = \eta_1 \cup \eta_2$  with  $\eta_1 \perp\!\!\!\perp \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}_i$  be weak repairs for  $\mathcal{I}$  and  $\eta_i$ , for  $i = 1, 2$ , such that all actions in  $\mathcal{U}_i$  affect a literal in the body of a rule in  $\eta_i$ .

Define  $\mathcal{U} = \mathcal{U}_1 \cup \mathcal{U}_2$ .

Then:

- $\mathcal{U}$  is a weak repair for  $\mathcal{I}$  and  $\eta$ ;
- if each  $\mathcal{U}_i$  is a repair, then so is  $\mathcal{U}$ ;
- if each  $\mathcal{U}_i$  is founded, then so is  $\mathcal{U}$ ;
- if each  $\mathcal{U}_i$  is justified, then so is  $\mathcal{U}$ .

# Independence vs. parallellization (IV)

The proof is similar.

## Independence vs. parallelization (IV)

The proof is similar.

This means that parallelization of the search does not add “new” (false) solutions.

# Computing independent sets of AICs

The previous results generalize to several independent sets of AICs.

# Computing independent sets of AICs

The previous results generalize to several independent sets of AICs.

In order to split a set  $\eta$  into independent sets, consider the relation  $\underline{\eta}^+$ .

# Computing independent sets of AICs

The previous results generalize to several independent sets of AICs.

In order to split a set  $\eta$  into independent sets, consider the relation  $\underline{\eta}^+$ . This is an equivalence relation.

# Computing independent sets of AICs

The previous results generalize to several independent sets of AICs.

In order to split a set  $\eta$  into independent sets, consider the relation  $\perp^+$ . This is an equivalence relation. The quotient set  $\eta / \perp^+$  is the finest partition of  $\eta$  in independent sets, and can be computed efficiently.

# Precedence

## Definition

AIC  $r_1$  *precedes* AIC  $r_2$ ,  $r_1 \prec r_2$ , if some action in the head of  $r_1$  affects a literal in the body of  $r_2$ .

# Precedence

## Definition

AIC  $r_1$  *precedes* AIC  $r_2$ ,  $r_1 \prec r_2$ , if some action in the head of  $r_1$  affects a literal in the body of  $r_2$ .

- Reflexive relation.

# Precedence

## Definition

AIC  $r_1$  *precedes* AIC  $r_2$ ,  $r_1 \prec r_2$ , if some action in the head of  $r_1$  affects a literal in the body of  $r_2$ .

- Reflexive relation.
- $\langle \eta / \approx, \preceq \rangle$  is a partial order, where  $\preceq$  is the transitive closure of  $\prec$  and  $\approx$  is the induced equivalence relation.

# Precedence

## Definition

AIC  $r_1$  *precedes* AIC  $r_2$ ,  $r_1 \prec r_2$ , if some action in the head of  $r_1$  affects a literal in the body of  $r_2$ .

- Reflexive relation.
- $\langle \eta / \approx, \preceq \rangle$  is a partial order, where  $\preceq$  is the transitive closure of  $\prec$  and  $\approx$  is the induced equivalence relation.

(Similar to stratified negation in logic programming. . .)

# Precedence vs. stratification (I)

## Theorem

*Let  $\eta_1, \eta_2 \in \eta / \approx$  with  $\eta_1 \prec \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}$  be a weak repair for  $\mathcal{I}$  and  $\eta_1 \cup \eta_2$ .*

# Precedence vs. stratification (I)

## Theorem

*Let  $\eta_1, \eta_2 \in \eta / \approx$  with  $\eta_1 \prec \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}$  be a weak repair for  $\mathcal{I}$  and  $\eta_1 \cup \eta_2$ .*

*Assume that every action in  $\mathcal{U}$  occurs in the head of a rule in  $\eta_1 \cup \eta_2$ .*

# Precedence vs. stratification (I)

## Theorem

*Let  $\eta_1, \eta_2 \in \eta / \approx$  with  $\eta_1 \prec \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}$  be a weak repair for  $\mathcal{I}$  and  $\eta_1 \cup \eta_2$ .*

*Assume that every action in  $\mathcal{U}$  occurs in the head of a rule in  $\eta_1 \cup \eta_2$ .*

*Define  $\mathcal{U}_i$  as the set of actions in  $\mathcal{U}$  in the head of a rule in  $\eta_i$ , for  $i = 1, 2$ .*

# Precedence vs. stratification (I)

## Theorem

*Let  $\eta_1, \eta_2 \in \eta / \approx$  with  $\eta_1 \prec \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}$  be a weak repair for  $\mathcal{I}$  and  $\eta_1 \cup \eta_2$ .*

*Assume that every action in  $\mathcal{U}$  occurs in the head of a rule in  $\eta_1 \cup \eta_2$ .*

*Define  $\mathcal{U}_i$  as the set of actions in  $\mathcal{U}$  in the head of a rule in  $\eta_i$ , for  $i = 1, 2$ .*

*Then:*

# Precedence vs. stratification (I)

## Theorem

Let  $\eta_1, \eta_2 \in \eta / \approx$  with  $\eta_1 \prec \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}$  be a weak repair for  $\mathcal{I}$  and  $\eta_1 \cup \eta_2$ .

Assume that every action in  $\mathcal{U}$  occurs in the head of a rule in  $\eta_1 \cup \eta_2$ .

Define  $\mathcal{U}_i$  as the set of actions in  $\mathcal{U}$  in the head of a rule in  $\eta_i$ , for  $i = 1, 2$ .

Then:

- $\mathcal{U}_1$  is a weak repair for  $\mathcal{I}$  and  $\eta_1$  and  $\mathcal{U}_2$  is a weak repair for  $\mathcal{I} \circ \mathcal{U}_1$  and  $\eta_2$ ;

# Precedence vs. stratification (I)

## Theorem

Let  $\eta_1, \eta_2 \in \mathcal{I} / \approx$  with  $\eta_1 \prec \eta_2$ ;  $\mathcal{I}$  be a database; and  $\mathcal{U}$  be a weak repair for  $\mathcal{I}$  and  $\eta_1 \cup \eta_2$ .

Assume that every action in  $\mathcal{U}$  occurs in the head of a rule in  $\eta_1 \cup \eta_2$ .

Define  $\mathcal{U}_i$  as the set of actions in  $\mathcal{U}$  in the head of a rule in  $\eta_i$ , for  $i = 1, 2$ .

Then:

- $\mathcal{U}_1$  is a weak repair for  $\mathcal{I}$  and  $\eta_1$  and  $\mathcal{U}_2$  is a weak repair for  $\mathcal{I} \circ \mathcal{U}_1$  and  $\eta_2$ ;
- if  $\mathcal{U}$  is founded/justified, then so is each  $\mathcal{U}_i$ .

## Precedence vs. stratification (II)

The proof is similar to the above.

## Precedence vs. stratification (II)

The proof is similar to the above.

This allows us to sequentialize the search for repairs.

## Precedence vs. stratification (II)

The proof is similar to the above.

This allows us to sequentialize the search for repairs.

However, some results do not hold:

## Precedence vs. stratification (II)

The proof is similar to the above.

This allows us to sequentialize the search for repairs.

However, some results do not hold:

- it may happen that  $\mathcal{U}$  is a repair, but  $\mathcal{U}_1$  and/or  $\mathcal{U}_2$  are not;

## Precedence vs. stratification (II)

The proof is similar to the above.

This allows us to sequentialize the search for repairs.

However, some results do not hold:

- it may happen that  $\mathcal{U}$  is a repair, but  $\mathcal{U}_1$  and/or  $\mathcal{U}_2$  are not;
- there may be a weak (founded, justified) repair  $\mathcal{U}_1$  for  $\mathcal{I}$  and  $\eta_1$  that is not a subset of any weak repair for  $\mathcal{I}$  and  $\eta_1 \cup \eta_2$ .

## Precedence vs. stratification (III)

### Theorem

*Let  $\eta_1$ ,  $\eta_2$  and  $\mathcal{I}$  be as before;  $\mathcal{U}_1$  be a weak repair for  $\mathcal{I}$  and  $\eta_1$ ;  $\mathcal{U}_2$  be a weak repair for  $\mathcal{I} \circ \mathcal{U}_1$  and  $\eta_2$ ; such that every action in  $\mathcal{U}_i$  occurs in the head of a rule in  $\eta_i$ .*

## Precedence vs. stratification (III)

### Theorem

*Let  $\eta_1$ ,  $\eta_2$  and  $\mathcal{I}$  be as before;  $\mathcal{U}_1$  be a weak repair for  $\mathcal{I}$  and  $\eta_1$ ;  $\mathcal{U}_2$  be a weak repair for  $\mathcal{I} \circ \mathcal{U}_1$  and  $\eta_2$ ; such that every action in  $\mathcal{U}_i$  occurs in the head of a rule in  $\eta_i$ .*

*Define  $\mathcal{U} = \mathcal{U}_1 \cup \mathcal{U}_2$ .*

## Precedence vs. stratification (III)

### Theorem

*Let  $\eta_1$ ,  $\eta_2$  and  $\mathcal{I}$  be as before;  $\mathcal{U}_1$  be a weak repair for  $\mathcal{I}$  and  $\eta_1$ ;  $\mathcal{U}_2$  be a weak repair for  $\mathcal{I} \circ \mathcal{U}_1$  and  $\eta_2$ ; such that every action in  $\mathcal{U}_i$  occurs in the head of a rule in  $\eta_i$ .*

*Define  $\mathcal{U} = \mathcal{U}_1 \cup \mathcal{U}_2$ .*

*Then:*

- *$\mathcal{U}$  is a weak repair for  $\mathcal{I}$  and  $\eta$ ;*
- *if each  $\mathcal{U}_i$  is a repair, then so is  $\mathcal{U}$ ;*
- *if each  $\mathcal{U}_i$  is founded/justified, then so is  $\mathcal{U}$ .*

## Precedence vs. stratification (III)

### Theorem

*Let  $\eta_1$ ,  $\eta_2$  and  $\mathcal{I}$  be as before;  $\mathcal{U}_1$  be a weak repair for  $\mathcal{I}$  and  $\eta_1$ ;  $\mathcal{U}_2$  be a weak repair for  $\mathcal{I} \circ \mathcal{U}_1$  and  $\eta_2$ ; such that every action in  $\mathcal{U}_i$  occurs in the head of a rule in  $\eta_i$ .*

*Define  $\mathcal{U} = \mathcal{U}_1 \cup \mathcal{U}_2$ .*

*Then:*

- *$\mathcal{U}$  is a weak repair for  $\mathcal{I}$  and  $\eta$ ;*
- *if each  $\mathcal{U}_i$  is a repair, then so is  $\mathcal{U}$ ;*
- *if each  $\mathcal{U}_i$  is founded/justified, then so is  $\mathcal{U}$ .*

The proof is similar.

# Outline

- 1 Integrity constraints
- 2 Active integrity constraints
- 3 Parallellization and stratification
- 4 Conclusions**

# What we achieved. . .

# What we achieved. . .

- Split a large problem in several smaller ones

# What we achieved. . .

- Split a large problem in several smaller ones
- Possibility of parallellization

# What we achieved. . .

- Split a large problem in several smaller ones
- Possibility of parallellization
- Stratification relation

... and what we still hope to do

# ... and what we still hope to do

- (More) practical evaluation

# ... and what we still hope to do

- (More) practical evaluation
- Prototype implementation

## ... and what we still hope to do

- (More) practical evaluation
- Prototype implementation
- Generalizations of AICs outside the database world

Thank you.