Department of Mathematics and Computer Science
University of Southern Denmark, Odense

December 15, 2011
LMF

DM528 – Weekly Note 7

Lectures in week 51

Monday, December 19

Kleinberg and Tardos, Algorithm Design:

- Section 13.9: Chernoff Bounds
- Section 13:10: Load Balancing
- Section 13.11: Packet Routing

Exercises for week 50

Monday December 19 / Tuesday, December 20

The following exercises are rephrased versions of exercises in Kleinberg and Tardos.

1. This exercise is about *coloring the vertices of a graph*. There are 3 colors available, and the aim is to maximize the number of edges that have endpoints with different colors.

   Give a randomized algorithm that colors the vertices such that at least $2/3$ of the edges have endpoints with different colors.

2. This exercise is about *contention resolution*. We have a system with n processes. Certain pairs of processes are in conflict, meaning that they need access to the same resource. Hence, such a pair cannot run at the same time. Assume that each proces is in conflict with exactly d other processes.

   Consider the following randomized algorithm for finding a largest possible set S of processes that can run simultaneously.
   Each process independently picks a random value. It chooses 1 with probability p and 0 with probability $1 - p$.
   If it chooses 1, and all conflicting processes choose 0, then the process enters the set S.

   What is the optimal value of p?
   What is the expected size of S for this value of p?

3. Consider a very simple *online auction system*:
   There are n bidding agents, and each agent has a bid. Assume that all bids are distinct from one another.
   The bidding agents appear in an order chosen uniformly at random, and at all times, the system maintains a variable b^* equal to the highest bid so far.

   What is the expected number of times that b^* is updated?

4. *Load balancing* algorithms for parallel or distributed systems seek to spread out collections of computing jobs over multiple machines. The jobs may come from diverse sources that cannot coordinate.

Suppose there are k machines, and k jobs show up for processing.
Consider the following randomized algorithm for load balancing.
Each job is assigned to one of the k machines independently at random (with each machine equally likely).

(a) Let $N(k)$ denote the expected number of machines that do not receive a job. Give a formula for

$$\lim_{k\to\infty} \frac{N(k)}{k}$$

(b) Assume that the machines cannot queue up jobs. Thus, if a machine receives more than one job, it rejects all but the first one. What is the expected total number of rejected jobs?

(c) Now assume that the machines can each buffer one job. Hence, a machine will reject jobs only if it receives more than two jobs.
Let $R(k)$ denote the expected number of rejected jobs.
Give a formula for

$$\lim_{k\to\infty} \frac{R(k)}{k}$$

- Exam DM528 January 2011 problem 5.

Tuesday December 20 / Wednesday, December 21

- The following two exercises are rephrased versions of exercises in Kleinberg and Tardos.

1. This exercise is about finding the median of a large set S of numbers.
We assume that all the numbers are distinct.
Let $n = |S|$.
A number x is an ε-approximate median, if
– at least $(\frac{1}{2} - \varepsilon)n$ of the numbers in S are smaller than x, and
– at least $(\frac{1}{2} - \varepsilon)n$ of the numbers in S are larger than x.
Consider the following randomized algorithm.
A random subset $S' \subseteq S$ is chosen, and the median of S' is returned.
Let $c = |S'|$. Show that c can be chosen independently of n such that, with probability at least 0.99, the element returned is an 0.05-approximate median.
2. Consider the following simple model of gambling in the presence of bad odds.
At the beginning, your net profit is 0.
You play for a sequence of n rounds. In each round, your net profit
– increases by 1 with probability $1/3$ and
– decreases by 1 with probability $2/3$.
It is allowed to have a negative net profit.
Show that the expected number of steps in which your net profit is positive can be upper-bounded by a constant, independent of n.

- Exam DM528 January 2010 problem 5.

Instruktorater

Hjælp dine medstuderende, dygtiggør dig selv og få penge for det. Søg i dag!

Der er ofte gode muligheder for at få et instruktorat, selvom man ikke er langt henne i studiet. Se nærmere information på

`http://www.jobs.sdu.dk/vis_stilling.php?id=6984&lang=da`

Instruktorer, der ifølge deres seneste ansættelseskontrakt allerede er ansat for foråret 2012, skal naturligvis ikke søge på stillingsopslaget. I stedet indleverer de ønsker om undervisning i foråret 2012, liste over tidligere undervisningserfaring, samt udskrift af eksamensprotokol til IMADA's sekretariat.

Der ydes hjælp til nye instruktorer i form af møder og diskussion i et mindre omfang.

Hvis du har spørgsmål, så henvend dig gerne på IMADA.

ANSØGNINGSFRIST: **21. december 2011 kl. 12.00** .