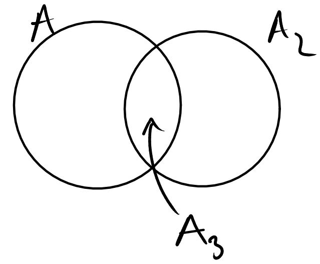
$\boxed{\text{DM549 feb. 2016}}$

Opgave 1

a) $B_1 = A_1$ (alle lige tal)



$B_2 = A_3$, ifølge Def. 4.1.1

$B_3 = A_2$, ——— " ———

$B_4 = \{2(k+2) \mid k \in \mathbb{Z}\} = A_1$

$B_5 = \{n \in \mathbb{Z} \mid 2|n \wedge 3|n\} = A_3$

$B_6 = A_2$, da $A_3 \subseteq A_2$

$B_7 = \{\dots, -9, -3, 3, 9, \dots\} \neq A_1, A_2, A_3$

$B_8 = A_2$, ifølge Def. 4.1.3

$B_9 = \mathbb{Z} \neq A_1, A_2, A_3$

$B_{10} = \emptyset \neq A_1, A_2, A_3$

$B_{11} = A_2$, da $\gcd(n,3) = 3 \iff 3|n$ (Def. 4.3.2)

b) A_1 er tællelig uendelig, da elementerne kan opstilles i en uendelig række:

$\dots, -4, -2, 0, 2, 4, \dots$

D.v.s. kardinaliteten er $\aleph_0$

Opgave 2:

a) $y = 4x^2 - 2 \iff x = \sqrt{\frac{y+2}{4}} = \frac{\sqrt{y+2}}{2}$

D.v.s.

$f^{-1}(x) = \frac{\sqrt{x+2}}{2}$

b) $(f \circ f)(x) = f(f(x)) = f(4x^2-2) = 4(4x^2-2)^2 - 2$

$= 4(16x^4 - 16x^2 + 4) - 2$

$= 64x^4 - 64x^2 + 14$

Opgave 3:

„$\Leftarrow$":

$$\begin{aligned} & n \text{ lige} \\ \Downarrow \quad & \\ & n = 2k, \quad k \in \mathbb{Z} \\ \Downarrow \quad & n^2 = 4k^2 = 2 \cdot 2k^2, \quad 2k^2 \in \mathbb{Z} \\ \Downarrow \quad & n^2 \text{ lige} \end{aligned}$$

„$\Rightarrow$" vises ved kontraposition:

$$\begin{aligned} & n \text{ ulige} \\ \Downarrow \quad & \\ & n = 2k+1, \quad k \in \mathbb{Z} \\ \Downarrow \quad & n^2 = 4k^2 + 4k + 1 = 2(2k^2+2k)+1, \quad 2k^2+2k \in \mathbb{Z} \\ \Downarrow \quad & n^2 \text{ ulige} \end{aligned}$$

Opgave 4:

a) $S_0 = (0+1)\cdot 2^0 = 1\cdot 1 = 1$

$S_1 = S_0 + (1+1)\cdot 2^1 = 1 + 2\cdot 2 = 5$

$S_2 = S_1 + (2+1)\cdot 2^2 = 5 + 3\cdot 4 = 17$

b) <u>Basis</u>: $n=0$

$1 = S_0 = 0\cdot 2^{0+1} + 1 = 1$ ✓

<u>Ind.ant.</u>: $n \geq 0$

$S_{n-1} = (n-1)\cdot 2^n + 1$

<u>Ind.skridt</u>:

$S_n = S_{n-1} + (n+1)2^n$, ifølge def. af S_n

$= (n-1)\cdot 2^n + 1 + (n+1)2^n$, ifølge ind.ant.

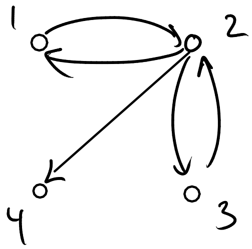
$= (n-1+n+1)\cdot 2^n + 1$

$= 2n\cdot 2^n + 1$

$= n\cdot 2^{n+1} + 1$

Opgave 5:

a)



b) Nej, R er hverken refleksiv, symmetrisk eller transitiv.

c) $R^2 = R \circ R = \{(1,1), (1,3), (1,4), (2,2), (3,1), (3,3), (3,4)\}$

$R^3 = R \circ R^2 = \{(1,2), (2,1), (2,3), (2,4), (3,2)\}$
$(= R)$

d)
$$(a,c) \in R^2$$
$$\Downarrow$$
$$\exists b : (a,b), (b,c) \in R$$
$$\Downarrow$$
$$\exists b : (b,a), (c,b) \in R, \quad \text{da } R \text{ er symmetrisk}$$
$$\Downarrow$$
$$(c,a) \in R^2$$

Opgave 6:

$$\lim_{n\to\infty} \frac{2n^2+2n+3}{n^2-3} = \lim_{n\to\infty} \frac{2+\frac{2}{n}+\frac{3}{n^2}}{1-\frac{3}{n^2}}$$

$$= \frac{2+0+0}{1-0}, \quad \text{ifølge regnereglerne s. 499 ø. i Adams.}$$

$$= 2$$