

DM538 – Weekly Note 14

Exercises, week 51

Thursday, December 13

Note that this session will take place in **U147**.

1. Recall that a family \mathcal{H} of hash functions is called *universal* if for any $u, v \in U$ with $u \neq v$,
$$\Pr(h(u) = h(v)) \leq \frac{1}{p}$$
when h is chosen uniformly at random from \mathcal{H} .

Similarly, a family \mathcal{H} of hash functions is called *2-universal* if

- for any $u_1, u_2, v_1, v_2 \in U$ with $u_1 \neq u_2, u_1 \neq v_1$, and $u_2 \neq v_2$,
- $$\Pr(h(u_1) = h(v_1) \wedge h(u_2) = h(v_2)) \leq \frac{1}{p^2}$$
- when h is chosen uniformly at random from \mathcal{H} .

Consider the universal class of hash functions defined on page 739. Is this class 2-universal?

2. Exam January 2012 Problem 2 a)
3. This exercise is about finding the *median* of a large set S of numbers. We assume that all the numbers are distinct. Let $n = |S|$.
A number x is an ε -approximate median, if
 - at least $(\frac{1}{2} - \varepsilon)n$ of the numbers in S are smaller than x , and
 - at least $(\frac{1}{2} - \varepsilon)n$ of the numbers in S are larger than x .

Consider the following randomized algorithm.

A random subset $S' \subseteq S$ is chosen, and the median of S' is returned.

Let $c = |S'|$. Show that c can be chosen independently of n such that, with probability at least 0.99, the element returned is an 0.05-approximate median.

Hint: Consider the elements of S in sorted order and let a and b be the smallest and largest element, respectively, among the middle 10% of the elements. Find an upper bound on the probability that either more than half of the sampled items are smaller than a or more than half of them are larger than b .

Spørgetime

Indtil videre er der flertal for at holde spørgetimen onsdag d. 23/1 kl 10:15, men der er stadig nogle, som ikke har stemt.