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DM825 - Introduction to Machine Learning

Sheet 12, Spring 2013

Exercise 1 – Bayesian Networks – Inference
Prove that inference in BN is #P-complete

Solution

Exercise 2 – Bayesian Networks – Inference
Figure shows a graphical model with conditional probabilities tables about whether or not you will panic at an exam based on whether or not the course was boring ("B"), which was the key factor you used to decide whether or not to attend lectures ("A") and revise doing the exercises after each lecture ("R").
You should use the model to make exact *inference* and answer the following queries:

- what is the probability that you will panic or not before the exam given that you attended the lectures and revised after each lecture?

  Solution

  From the CPT, 0.

- what is the probability that you will panic or not before the exam? **Solution**

$$\begin{aligned}
\Pr(p) &= \sum_{b,r,a} \Pr(b,r,a,p) \\
&= \sum_{b,r,a} \Pr(b)\Pr(r|b)\Pr(a|b)\Pr(p|r,a) \\
&= \sum_{b} \Pr(b) \sum_{r,a} \Pr(r|b)\Pr(a|b)\Pr(p|r,a) \\
&= 0.5 \cdot (0.3 \cdot 0.1 \cdot 0 + 0.3 \cdot 0.9 \cdot 0.8 + 0.7 \cdot 0.1 \cdot 0.6 + 0.7 \cdot 0.9 \cdot 1) \\
&\quad +0.5 \cdot (0.8 \cdot 0.5 \cdot 0 + \cdot 0.8 \cdot 0.5 \cdot 0.8 + 0.2 \cdot 0.5 \cdot 0.6 + 0.2 \cdot 0.5 \cdot 1) \\
&= 0.684
\end{aligned}$$

- your teacher saw you panicking at the exam and he wants to work out from the model the reason for that. Was it because you did not come to the lecture or because you did not revise? **Solution**

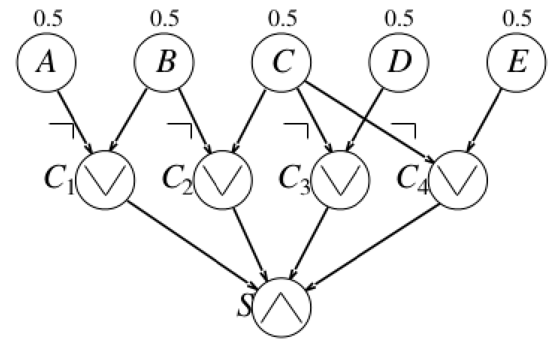


Figure S14.3 A Bayesian network corresponding to a SAT problem.

14.8 Consider a SAT problem such as the following:

$$(\neg A \vee B) \wedge (\neg B \vee C) \wedge (\neg C \vee D) \wedge (\neg C \vee \neg D \vee E)$$

The idea is to encode this as a Bayes net, such that doing inference in the Bayes net gives the answer to the SAT problem.

a. Figure S14.3 shows the Bayes net corresponding to this SAT problem. The general construction method is as follows:

- The root nodes correspond to the logical variables of the SAT problem. They have a prior probability of 0.5.
- Each clause C_i is a node. Its parents are the variables in the clause. The CPT is deterministic and implements the disjunction given in the clause. (Negative literals in the clause are indicated by negation symbols on the links in the figure.)
- A single sentence node S has all the clauses as parents and a CPT that implements deterministic conjunction.

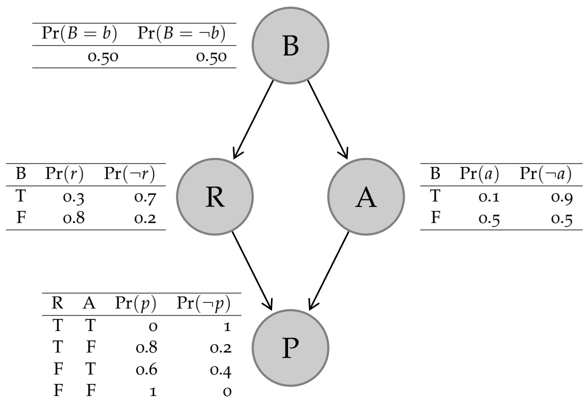
It is clear that $P(S) > 0$ iff the SAT problem is satisfiable. Hence, we have reduced SAT to Bayes net inference. Since SAT is NP-complete, we have shown that Bayes net inference is NP-hard (even without evidence).

b. The prior probability of each complete assignment is 2^{-n}. $P(S)$ is therefore $K \cdot 2^{-n}$ where K is the number of satisfying assignments. Hence, we can count the number of satisfying assignments by computing $P(S) \cdot 2^n$. This amounts to a reduction of the problem of counting satisfying assignments to Bayes net inference; since the former is #P-complete, the latter is #P-hard.

$$\begin{aligned}
\Pr(r|p) &= \frac{\Pr(p|r)\Pr(r)}{\Pr(p)} \\
&= \frac{\sum_{b,a}\Pr(b,a,r,p)}{\Pr(p)} \\
&= \frac{0.5\cdot(0.3\cdot 0.1\cdot 0+0.3\cdot 0.9\cdot 0.8)+0.5\cdot(0.8\cdot 0.5\cdot 0+0.8\cdot 0.5\cdot 0+0.8\cdot 0.5\cdot 0.8)}{\Pr(p)} \\
&= \frac{0.268}{0.684} = 0.3918
\end{aligned}$$

$$\begin{aligned}
\Pr(r|a) &= \frac{\Pr(p|a)\Pr(a)}{\Pr(p)} \\
&= \frac{0.144}{0.684} = 0.2105
\end{aligned}$$

Repeat the inference in the last two queries by means of stochastic inference implementing in R the Prior-Sample, Rejection-Sampling, Likelihood-weighting and Gibbs-Sampling Markov Chain Monte Carlo.



$\Pr(B=b)$	$\Pr(B=\neg b)$
0.50	0.50

B	$\Pr(r)$	$\Pr(\neg r)$
T	0.3	0.7
F	0.8	0.2

B	$\Pr(a)$	$\Pr(\neg a)$
T	0.1	0.9
F	0.5	0.5

R	A	$\Pr(p)$	$\Pr(\neg p)$
T	T	0	1
T	F	0.8	0.2
F	T	0.6	0.4
F	F	1	0

Figure 1: The graphical model of exercise 1. Lower-case letter indicate the outcome that the upper-case letter can take.