LOCAL SEARCH METHODS
APPLICATIONS AND ENGINEERING

Lecture 10

Vehicle Routing

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Outline

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Problem Definition

Vehicle Routing problems concern the distribution of goods between depots and final users

Examples are: solid waste collection, street cleaning, schools bus routing, dial-a-ride systems, transportation of handicapped persons, routing of salespeople and maintenance unit.

General Formulation

Input: Vehicles, depots, drivers, road network, costs and customers requirements.
Task: Find a collection of routes, each performed by a single vehicle, and starting and ending at the depot, such that:

- requirement of customers are fulfilled,
- operational constraints are satisfied and
- a global transportation cost is minimized.

Road Network

- represented by a directed or undirected (complete) graph
- travel costs and travel times on the arcs obtained by *shortest paths*

Customers

- vertices of the graph
- collection or delivery demands
- time windows for service
- loading and unloading times
- subset of vehicles that can serve them

Vehicle fleets

- fix costs associated to the use of a vehicle
- subsets of arcs traversed by the vehicle
- a-priori partition of customers
- capacity
- home depot in multi-depot systems
- working contracts of the drivers

Operational Constraints

- current load cannot exceed vehicle capacity
- customers in the route can require only delivery or collection of goods
- customers must be visited within their time windows
- a vehicle must be used in the working periods of the drivers
- precedence constraints on the customers

Objectives

- minimization of global transportation cost (variable + fixed costs)
- minimization of the number of vehicles
- balancing of the routes for travel time and vehicle load
- minimization of penalties for un-served customers

Vehicle Routing Models

- Capacited and Distance Constrained VRP (CVRP and DCVRP)
- VRP with Time Windows (VRPTW)
- VRP with Backhauls (VRPB)
- VRP with Pickup and Delivery (VRPPD)

Capacited Vehicle Routing (CVRP)

Input:

- complete graph $G(V, A)$, where $V = \{0, \ldots, n\}$
- vertices $i = 1, \ldots, n$ are customer deliveries, not splittable
- vertex $i = 0$ is depot (one single!)
- arc/edges have associated a cost c_{ij} ($c_{ik} + c_{kj} \geq c_{ij} \qquad \forall\, i, j \in V$)
- costumers have associated a non-negative demand d_i
- a set of K identical vehicles with capacity C
  ($d_i \leq C$, and $K \geq K_{min}$ where K_{min} is the number of bins in the associated *Bin Packing Problem*),

Task:
Find collection of K circuits with minimum cost, defined as the sum of the costs of the arcs of the circuits and such that:

- each circuit visit the depot vertex
- each customer vertex is visited by exactly one circuit; and
- the sum of the demands of the vertices visited by a circuit does not exceed the vehicle capacity C.

Variants:

- fixed costs associated with the circuits summed in the total cost, thus the minimization involves the number of circuits.
- different vehicles
- total duration of a route cannot exceed T associated with each vehicle
- service times s_i associated with vertices or added to the travel times of the arcs: $t'_{ij} = t_{ij} + s_i/2 + s_j/2$

Generally $c_{ij} = t_{ij}$, then minimizing the cost corresponds to min the length and, with services, the duration.

Vehicle Routing with Time Windows (VRPTW)

It is an extension of the CVRP in which:

- each vertex is also associated with a time interval $[a_i, b_j]$.
- each arc is associated with a travel time t_{ij}
- each vertex is associated with a service time s_i

Task:
Find a collection of K simple circuits with minimum costs, such that:

- each circuit visit the depot vertex
- each customer vertex is visited by exactly one circuit; and
- the sum of the demands of the vertices visited by a circuit does not exceed the vehicle capacity C.
- for each customer i, the service starts within the time windows $[a_i, b_i]$ and the vehicle stops for s_i time instants.

Note:
Typically allowed to wait until a_i in case of early arrive
Time windows induce orientation of the graph, hence VRPTW is an asymmetric problem.

Related Versions

- Minimize number of routes
- Minimize hierarchical objective function
- Makespan Problem with Time Windows (MPTW)
  minimizing the completion time
- Delivery Man Problem with Time Windows (DMPTW)
  minimizing the sum of customers waiting times
- Multiple TSPTW (mTSPTW) when multiple salesmen
  minimizing vehicle routes cost
- Vehicle Routing Problem with Time Windows (VRPTW)
  constraints also on vehicle capacity

Vehicle Routing with Backhauls (VRPB)

It is an extension of the CVRP in which:

- customers are partitioned in two subsets: $L = \{1, \dots, n\}$ Lineahaul customers (deliveries) and $B = \{n+1, \dots, n+m\}$ Backhaul customers (pickups)
- precedence constraints: in a same route customers from L must be served before customers from B

Task:
Find a collection of K simple circuits with minimum costs, such that:

- each circuit visit the depot vertex
- each customer vertex is visited by exactly one circuit; and
- the sum of the demands of the vertices visited by a circuit does not exceed the vehicle capacity C.
- in any circuit all the linehaul customers precede the backhaul customers, if any.

Note:
$K > \max\{K_L, K_B\}$ Time Windows constraints can be present (VRPBWT).

Vehicle Routing with Pickup and Delivery (VRPPD)

It is an extension of the CVRP in which:

- each customer i is associated with quantities d_i and p_i to be delivered and picked up, resp.
- for each customer i, O_i denotes the vertex that is the origin of the delivery demand and D_i denotes the vertex that is the destination of the pickup demand

Task:
Find a collection of K simple circuits with minimum costs, such that:

- each circuit visit the depot vertex
- each customer vertex is visited by exactly one circuit; and
- the current load of the vehicle along the circuit must be non-negative and may never exceed C
- for each customer i, the customer O_i when different from the depot, must be served in the same circuit and before customer i
- for each customer i, the customer D_i when different from the depot, must be served in the same circuit and after customer i

Note:
$K > \max\{K_L, K_B\}$

Outline

CVRP

Construction Heuristics

- Nearest neighbors
- Savings heuristics (Clarke and Wright)
- Insertion heuristics
- Route-second cluster-first
- Cluster-first route-second
  - Sweep algorithm
  - Generalized assignment
  - Location based heuristic
  - Petal algorithm

Perturbative Search

- Solution representation: sets of integer sequences, one per route
- Neighborhoods structures:
  - intra-route: 2-opt, 3-opt
  - inter-routes: λ-interchange, relocate, exchange, CROSS, ejection chains, GENI

Metaheuristics

Taburoute

Step 1: (Initialization) Generate $\lceil\sqrt{n}/2\rceil$ initial solutions and perform tabu search on $W' \subset W = V \setminus \{0\}$ ($|W'| \approx 0.9|W|$) up to 50 idle iterations.

Step 2: (Improvement) Starting with the best solution observed in Step 1 perform tabu search on $W' \subset W = V \setminus \{0\}$ ($|W'| \approx 0.9|W|$) up to $50n$ idle iterations.

Step 3: (Intensification) Starting with the best solution observed in Step 1, perform tabu search up to 50 idle iterations. Here W' is the set of the $\lceil|V|/2\rceil$ vertices that have been most often moved in Steps 1 and 2.

Outline

1. Vehicle Routing

2. CVRP

3. VRPTW

VRPTW

Construction Heuristics

Extensions of those for CVRP. Studied by Solomon (1987).

- Savings heuristics (Clarke and Wright)
- Time-oriented nearest neighbors
- Insertion heuristics
- Time-oriented sweep heuristic

Perturbative Search

- Solution representation: sets of integer sequences, one per route
- Neighborhoods structures:
  - intra route: 2-opt, 2H-opt, 3-opt (bad), or-opt (good),
  - inter routes: 2-opt* (good), λ-interchange, relocate, exchange, CROSS

Time-Oriented Nearest-Neighbor

- Add the unrouted node "closest" to the depot or the last node added without violating feasibility
- Metric for "closest":

$$d_{ij} = \delta_1 c_{ij} + \delta_2 T_{ij} + \delta_3 v_{ij}$$

c_{ij} geographical distance
T_{ij} time distance
v_{ij} urgency to serve j

Insertion Heuristics

Step 1: Compute for each unrouted costumer u the *best feasible position* in the route:
$c_1(i(u), u, j(u)) = \min_{p=1,\ldots,m} \{c_1(i_{p-1}, u, i_p)\}$

Step 2: Compute for each unrouted customer u which can be feasibly inserted: $c_2(i, u, j) = \lambda d_{0u} - c_1(i, u, j)$

Step 3: Insert the customer u^* from Step 2 that maximize c_2.

Best specific criteria: max the benefit of servicing a node on a partial route rather than on a direct route

Metaheuristics

A Tabu Search for VRPTW, Potvin (1996)

- **Initial solution**: Solomon's insertion heuristic
- **Neighborhood**: or-opt and 2-opt* (in VNS fashion or neighborhood union)
  speed up in or-opt: i is moved between j and $j+q$ if i is one of the h nearest neighbors
- **Step**: best improvement
- **Tabu Criteria**: forbidden to reinsert edges which were recently removed
- **Tabu length**: fixed
- **Aspiration criteria**: tabu move is overridden if an overall best is reached
- **End criteria**: number of iterations without improvements