

LOCAL SEARCH METHODS

APPLICATIONS AND ENGINEERING

Lecture 10

Vehicle Routing

Marco Chiarandini

Outline

1. Vehicle Routing
Introduction
2. CVRP
3. VRPTW

Outline

1. Vehicle Routing

2. CVRP

3. VRPTW

Problem Definition

Vehicle Routing problems concern the distribution of goods between depots and final users

Examples are: solid waste collection, street cleaning, schools bus routing, dial-a-ride systems, transportation of handicapped persons, routing of salespeople and maintenance unit.

General Formulation

Input: Vehicles, depots, drivers, road network, costs and customers requirements.

Task: Find a collection of routes, each performed by a single vehicle, and starting and ending at the depot, such that:

- ▶ requirement of customers are fulfilled,
- ▶ operational constraints are satisfied and
- ▶ a global transportation cost is minimized.

Road Network

- ▶ represented by a directed or undirected (complete) graph
- ▶ travel costs and travel times on the arcs obtained by *shortest paths*

Customers

- ▶ vertices of the graph
- ▶ collection or delivery demands
- ▶ time windows for service
- ▶ loading and unloading times
- ▶ subset of vehicles that can serve them

Vehicle fleets

- ▶ fix costs associated to the use of a vehicle
- ▶ subsets of arcs traversed by the vehicle
- ▶ a-priori partition of customers
- ▶ capacity
- ▶ home depot in multi-depot systems
- ▶ working contracts of the drivers

Operational Constraints

- ▶ current load cannot exceed vehicle capacity
- ▶ customers in the route can require only delivery or collection of goods
- ▶ customers must be visited within their time windows
- ▶ a vehicle must be used in the working periods of the drivers
- ▶ precedence constraints on the customers

Objectives

- ▶ minimization of global transportation cost (variable + fixed costs)
- ▶ minimization of the number of vehicles
- ▶ balancing of the routes for travel time and vehicle load
- ▶ minimization of penalties for un-served customers

Vehicle Routing Models

- ▶ Capacited and Distance Constrained VRP (CVRP and DCVRP)
- ▶ VRP with Time Windows (VRPTW)
- ▶ VRP with Backhauls (VRPB)
- ▶ VRP with Pickup and Delivery (VRPPD)

Capacited Vehicle Routing (CVRP)

Input:

- ▶ complete graph $G(V, A)$, where $V = \{0, \dots, n\}$
- ▶ vertices $i = 1, \dots, n$ are customer deliveries, not splittable
- ▶ vertex $i = 0$ is depot (one single!)
- ▶ arc/edges have associated a cost c_{ij} ($c_{ik} + c_{kj} \geq c_{ij} \quad \forall i, j \in V$)
- ▶ costumers have associated a non-negative demand d_i
- ▶ a set of K identical vehicles with capacity C
($d_i \leq C$, and $K \geq K_{min}$ where K_{min} is the number of bins in the associated *Bin Packing Problem*),

Task:

Find collection of K circuits with minimum cost, defined as the sum of the costs of the arcs of the circuits and such that:

- ▶ each circuit visit the depot vertex
- ▶ each customer vertex is visited by exactly one circuit; and
- ▶ the sum of the demands of the vertices visited by a circuit does not exceed the vehicle capacity C .

Variants:

- ▶ fixed costs associated with the circuits summed in the total cost, thus the minimization involves the number of circuits.
- ▶ different vehicles
- ▶ total duration of a route cannot exceed T associated with each vehicle
- ▶ service times s_i associated with vertices or added to the travel times of the arcs: $t'_{ij} = t_{ij} + s_i/2 + s_j/2$

Generally $c_{ij} = t_{ij}$, then minimizing the cost corresponds to min the length and, with services, the duration.

Vehicle Routing with Time Windows (VRPTW)

It is an extension of the CVRP in which:

- ▶ each vertex is also associated with a time interval $[a_i, b_j]$.
- ▶ each arc is associated with a travel time t_{ij}
- ▶ each vertex is associated with a service time s_i

Task:

Find a collection of K simple circuits with minimum costs, such that:

- ▶ each circuit visit the depot vertex
- ▶ each customer vertex is visited by exactly one circuit; and
- ▶ the sum of the demands of the vertices visited by a circuit does not exceed the vehicle capacity C .
- ▶ for each customer i , the service starts within the time windows $[a_i, b_i]$ and the vehicle stops for s_i time instants.

Note:

Typically allowed to wait until a_i in case of early arrive

Time windows induce orientation of the graph, hence VRPTW is an asymmetric problem.

Related Versions

- ▶ Minimize number of routes
- ▶ Minimize hierarchical objective function
- ▶ Makespan Problem with Time Windows (MPTW)
minimizing the completion time
- ▶ Delivery Man Problem with Time Windows (DMPTW)
minimizing the sum of customers waiting times
- ▶ Multiple TSPTW (mTSPTW) when multiple salesmen
minimizing vehicle routes cost
- ▶ Vehicle Routing Problem with Time Windows (VRPTW)
constraints also on vehicle capacity

Vehicle Routing with Backhauls (VRPB)

It is an extension of the CVRP in which:

- ▶ customers are partitioned in two subsets: $L = \{1, \dots, n\}$ Linehaul customers (deliveries) and $B = \{n + 1, \dots, n + m\}$ Backhaul customers (pickups)
- ▶ precedence constraints: in a same route customers from L must be served before customers from B

Task:

Find a collection of K simple circuits with minimum costs, such that:

- ▶ each circuit visit the depot vertex
- ▶ each customer vertex is visited by exactly one circuit; and
- ▶ the sum of the demands of the vertices visited by a circuit does not exceed the vehicle capacity C .
- ▶ in any circuit all the linehaul customers precede the backhaul customers, if any.

Note:

$K > \max\{K_L, K_B\}$ Time Windows constraints can be present (VRPBWT).

Vehicle Routing with Pickup and Delivery (VRPPD)

It is an extension of the CVRP in which:

- ▶ each customer i is associated with quantities d_i and p_i to be delivered and picked up, resp.
- ▶ for each customer i , O_i denotes the vertex that is the origin of the delivery demand and D_i denotes the vertex that is the destination of the pickup demand

Task:

Find a collection of K simple circuits with minimum costs, such that:

- ▶ each circuit visit the depot vertex
- ▶ each customer vertex is visited by exactly one circuit; and
- ▶ the current load of the vehicle along the circuit must be non-negative and may never exceed C
- ▶ for each customer i , the customer O_i when different from the depot, must be served in the same circuit and before customer i
- ▶ for each customer i , the customer D_i when different from the depot, must be served in the same circuit and after customer i

Note:

$$K > \max\{K_L, K_B\}$$

Outline

1. Vehicle Routing
2. CVRP
3. VRPTW

Construction Heuristics

- ▶ Nearest neighbors
- ▶ Savings heuristics (Clarke and Wright)
- ▶ Insertion heuristics
- ▶ Route-second cluster-first
- ▶ Cluster-first route-second
 - ▶ Sweep algorithm
 - ▶ Generalized assignment
 - ▶ Location based heuristic
 - ▶ Petal algorithm

Perturbative Search

- ▶ Solution representation: sets of integer sequences, one per route
- ▶ Neighborhoods structures:
 - ▶ intra-route: 2-opt, 3-opt
 - ▶ inter-routes: λ -interchange, relocate, exchange, CROSS, ejection chains, GENI

Metaheuristics

Taburoute

- Step 1: (Initialization) Generate $\lceil \sqrt{n}/2 \rceil$ initial solutions and perform tabu search on $W' \subset W = V \setminus \{0\}$ ($|W'| \approx 0.9|W|$) up to 50 idle iterations.
- Step 2: (Improvement) Starting with the best solution observed in Step 1 perform tabu search on $W' \subset W = V \setminus \{0\}$ ($|W'| \approx 0.9|W|$) up to $50n$ idle iterations.
- Step 3: (Intensification) Starting with the best solution observed in Step 1, perform tabu search up to 50 idle iterations. Here W' is the set of the $\lceil |V|/2 \rceil$ vertices that have been most often moved in Steps 1 and 2.

Outline

1. Vehicle Routing
2. CVRP
3. VRPTW

Construction Heuristics

Extensions of those for CVRP. Studied by Solomon (1987).

- ▶ Savings heuristics (Clarke and Wright)
- ▶ Time-oriented nearest neighbors
- ▶ Insertion heuristics
- ▶ Time-oriented sweep heuristic

Perturbative Search

- ▶ Solution representation: sets of integer sequences, one per route
- ▶ Neighborhoods structures:
 - ▶ intra route: 2-opt, 2H-opt, 3-opt (bad), or-opt (good),
 - ▶ inter routes: 2-opt* (good), λ -interchange, relocate, exchange, CROSS

Time-Oriented Nearest-Neighbor

- ▶ Add the unrouted node “closest” to the depot or the last node added without violating feasibility
- ▶ Metric for “closest”:

$$d_{ij} = \delta_1 c_{ij} + \delta_2 T_{ij} + \delta_3 v_{ij}$$

c_{ij} geographical distance

T_{ij} time distance

v_{ij} urgency to serve j

Insertion Heuristics

Step 1: Compute for each unrouted customer u the *best feasible position* in the route:

$$c_1(i(u), u, j(u)) = \min_{p=1, \dots, m} \{c_1(i_{p-1}, u, i_p)\}$$

Step 2: Compute for each unrouted customer u which can be feasibly inserted: $c_2(i, u, j) = \lambda d_{0u} - c_1(i, u, j)$

Step 3: Insert the customer u^* from Step 2 that maximize c_2 .

Best specific criteria: max the benefit of servicing a node on a partial route rather than on a direct route

Metaheuristics

A Tabu Search for VRPTW, Potvin (1996)

- ▶ **Initial solution:** Solomon's insertion heuristic
- ▶ **Neighborhood:** or-opt and 2-opt* (in VNS fashion or neighborhood union)
 - speed up in or-opt: i is moved between j and $j + q$ if i is one of the h nearest neighbors
- ▶ **Step:** best improvement
- ▶ **Tabu Criteria:** forbidden to reinsert edges which were recently removed
- ▶ **Tabu length:** fixed
- ▶ **Aspiration criteria:** tabu move is overridden if an overall best is reached
- ▶ **End criteria:** number of iterations without improvements