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DM86 Local Search Methods – Weekly Notes

Week 1, Spring 2006

Lecture February 2

Details of the course DM/DMP86 were discussed. The reference page for any communication is the course homepage:

`http://www.imada.sdu.dk/~marco/Teaching/DMP86/`

The final assessment will consist of two parts, a course project and an oral exam. The modalities of the evaluation process are described under the link Evaluation from the homepage of the course.

The students are also invited to take vision of the requirements for the project under the link Exam Project from the course homepage. The selection of a combinatorial optimization problem to study in the project must be done rather soon so to leave enough time for the elaboration of the project. Link with ideas for helping in the selection are given in the Exam Project section.

The course will continue with one lecture per week. Nevertheless between week 7 and 10 there could be an additional lecture per week in order to anticipate the end of the course and leave more time to working on the project.

The text book is available for consultation at the IMADA library. Pointers to further relevant literature will be added in the section Literature.

Students who do not have knowledge of the statistical environment **R** are invited to download the software and take a look to the manuals. Links to the Software are available in the Course Material section, and to the relevant documentation in the Literature section.

The course has a mailing list to easy the diffusion of announcements and discussion. A forum has also been opened within the e-learn system for anonymous constructive criticism.

The rest of the lecture was aimed at the introduction and definition of the terminology which will be used in the course and the presentation of basic concepts. The following topics were treated.

- Combinatorial problems
- Three toy problems: traveling salesman, vertex coloring and scheduling
- Issues from complexity theory
- Solution methods for combinatorial problems: exact vs approximate
- Local search methods: nomenclature and components

The slides of the lecture are available for downloading.

Exercises

Exercise 1

Give the definitions of the following concepts

- Combinatorial problem and problem instance
- Complexity classes $\mathcal{P}$, $\mathcal{NP}$, $\mathcal{NP}$-complete, $\mathcal{NP}$-hard
- Approximation ratio
- Local search methods

Recall the components of Local search algorithms.

Exercise 2

In the Constraint Satisfaction Problem we are given a set of variables $V_1, V_2, \ldots, V_n$ with non empty domain $D_1, D_2, \ldots, D_n$ of possible values and a set of constraints $C_1, C_2, \ldots, C_m$ that involve some subset of variables and specify the allowable combinations of values for that subset. State the different decision and optimization variants for this problem. Which problem encountered at the lecture can be modeled as a Constraint Satisfaction Problem?

Exercise 3

Design one or more construction heuristics for the three toy problems encountered at the lecture.

Exercise 4

Show analytically how polynomial and exponential curves vary when passing from linear to semilogarithmic and logarithmic (log-log) scale systems. Use **R** to plot some of these curves in the three scale systems.

Exercise 5

Consider the following argument. For the Euclidean TSP, given an arbitrary approximation ratio $r > 1$, there exists a deterministic algorithm that achieves that ratio in polynomial run-time w.r.t. the number of vertices, n. Hence, the associated decision problems for arbitrary solution quality bounds can be solved by a deterministic algorithm with run-time polynomial in n, which implies that the search variant of the Euclidean TSP is also efficiently solvable. This conclusion is in direct contradiction with the known result that the Euclidean TSP is $\mathcal{NP}$-hard. Where is the argument flawed?

Exercise 6

Learn more about two of the toy problems introduced by consulting the following important on-line references:

- Traveling salesman problem: `http://www.tsp.gatech.edu/`
- Graph coloring problem: `http://www.cs.ualberta.ca/~joe/Coloring/` and `http://mathworld.wolfram.com/topics/GraphColoring.html` (for graph coloring related terms).