

## DM86 Local Search Methods – Weekly Notes

Week 10, Spring 2006

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### Lecture April 6

We reviewed the class of Vehicle Routing problems. After a general definition, we focused on Capacitated Vehicle Routing Problem (CVRP), Vehicle Routing Problem with Time Windows (VRPTW), Vehicle Routing Problems with Backhaul (VRPB) and Vehicle Routing Problems with Pickup and Delivery (VRPPD). This introductory part is described in the book by Toth and Vigo [3] sections 1.1-1.2.4 (photocopies have been provided). We then concentrated our attention to the CVRP and VRPTW. For the two problems we described the following construction heuristics (or extensions thereof): nearest neighbors, savings algorithms, insertion heuristics and the two-phases heuristics: route-first clustering-second and clustering-first route-second (sweep, generalized assignment, location based, and petal algorithms). As far as perturbative search is concerned, we described the possible neighborhood operators for intra-route changes: 2-opt, 2H-opt, 3-opt, or-opt; and for inter-route changes:  $\lambda$ -interchange, relocation, exchange, cross, 2-opt\*. This part is described on the survey by Bräysy [1] (for the exam, only the notions given at the lecture are necessary from this article). On the VRPTW we defined few more introductory concepts such as the push forward time. This is explained in Section 1 of the article of by Solomon [2].

### References

- [1] O. Bräysy. Vehicle routing problem with time windows, part I: Route construction and local search algorithms. *Transportation Science*, 39(1):104–118, 2005.
- [2] Marius M. Solomon. Algorithms for the vehicle routing and scheduling problems with time window constraints. *Operations Research*, 35(2):254–266, 1987.
- [3] Paolo Toth and Daniele Vigo, editors. *The Vehicle Routing Problem*. SIAM Monographs on Discrete Mathematics and Applications, 2002.

### Exercises

#### Exercise 1

For each construction heuristic on the CVRP and VRPTW presented at the lecture state whether it entails a fixed number of routes  $K$ , determined a priori or whether the heuristic can be used also for determining an initial value for  $K$ .

#### Exercise 2

In the VRPTW some of the arcs in the complete graph, representing the road network, can be eliminated due to the windows constraints. State which are the conditions for this arc elimination.

**Exercise 3**

In the TSP the local optimality of a solution in a neighborhood with  $k$ -opt operator can be determined in  $O(n^k)$ . On the VRPTW it is reasonable to expect a complexity of  $O(n^{k+1})$  because for each move the feasibility of all the customers routed after the changes must be ascertained. Show instead that, by using preprocessing, global variables, updating mechanisms and lexicographic search strategy, this complexity can be reduced to  $O(n^k)$ .

**Exercise 4**

Describe and formalize the problem that you chose for the Exam Project. Consult the lecturer to resolve any related questions.