DM502
Programming A

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Python & Linux Install Party

- Tomorrow (Tuesday, September 12) from 10 – 14
- Fredagsbar (area south of Kantine II)

- Participants are those
  - who want Python (& Swampy) on their computer,
  - who want Linux on their computer,
  - who want some study-related software on their computer,
  - who have problems with some study related software, or
  - who just like to hang out and help other people!

- drinks and some snacks will be provided by IMADA!

RECURSION:
SEE RECURSION

Recursion is "Complete"

- so far we know:
  - values of type integer, float, string
  - arithmetic expressions
  - (recursive) function definitions
  - (recursive) function calls
  - conditional execution
  - input/output

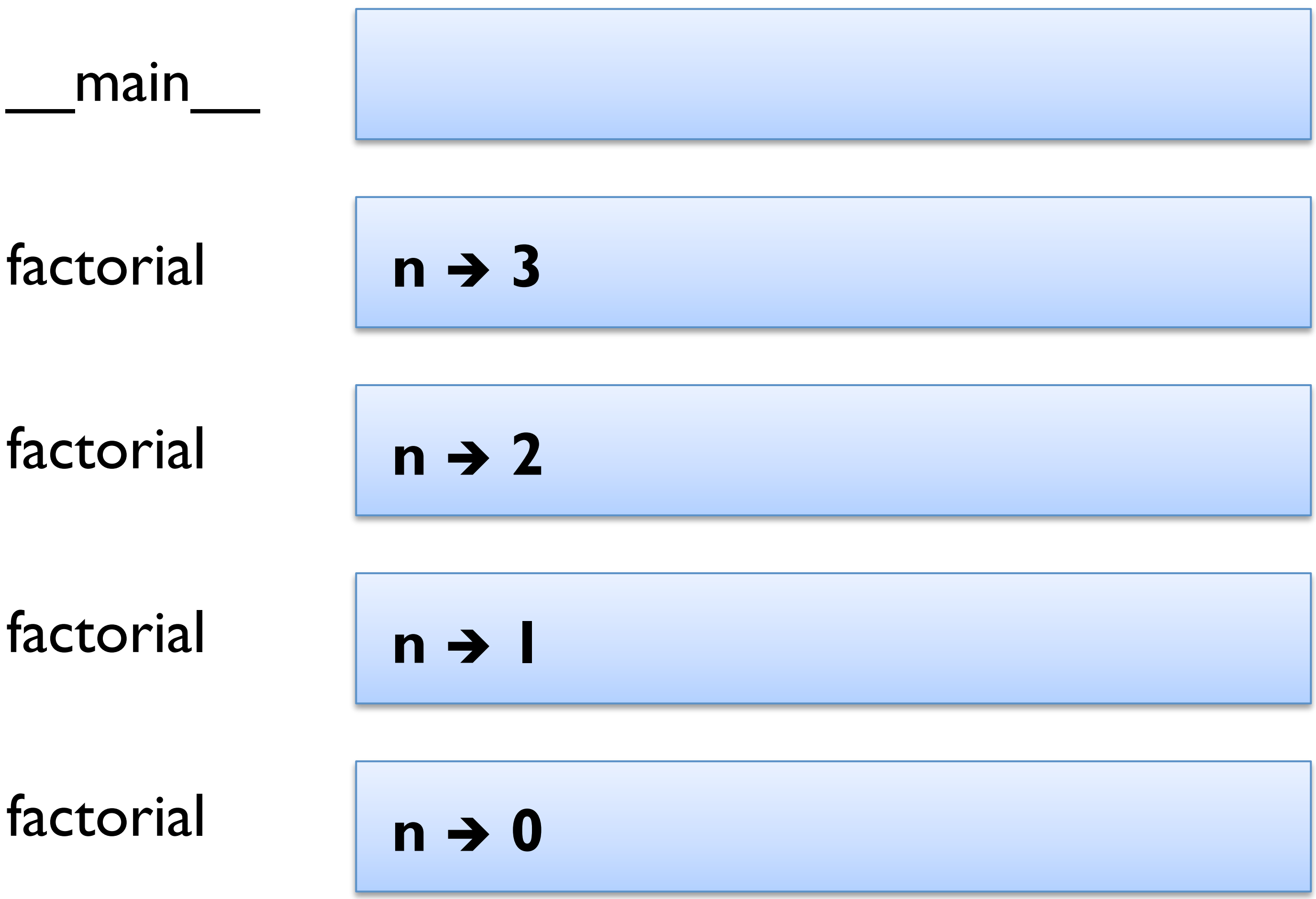
- ALL possible programs can be written using these elements!
- we say that we have a "Turing complete" language

Factorial

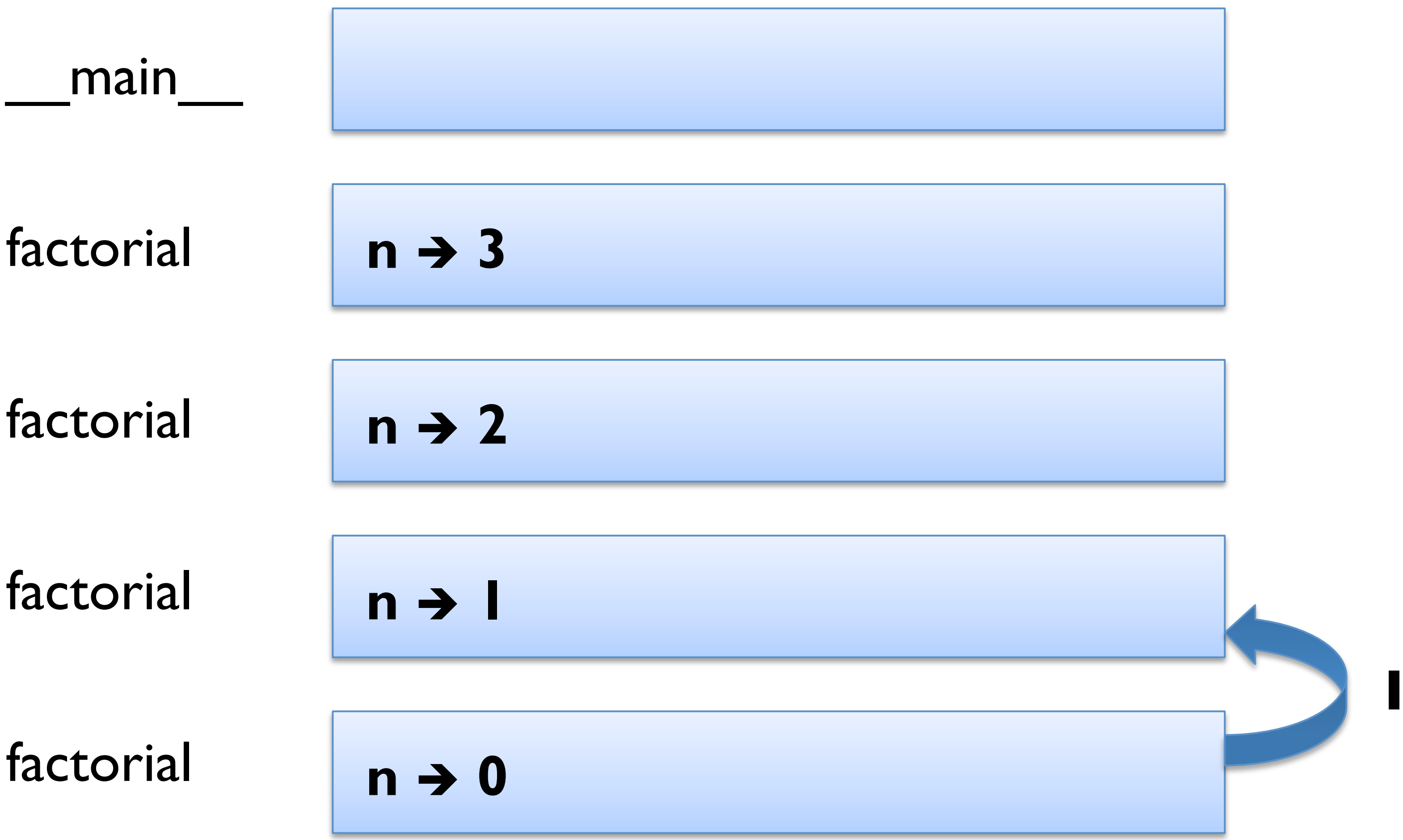
- in mathematics, the factorial function is defined by
  - 0! = 1
  - n! = n * (n-1)!
- such *recursive* definitions can trivially be expressed in Python
- Example:

```
def factorial(n):
    if n == 0:
        return 1
    recurse = factorial(n-1)
    result = n * recurse
    return result
x = factorial(3)
```

Stack Diagram for Factorial



Stack Diagram for Factorial



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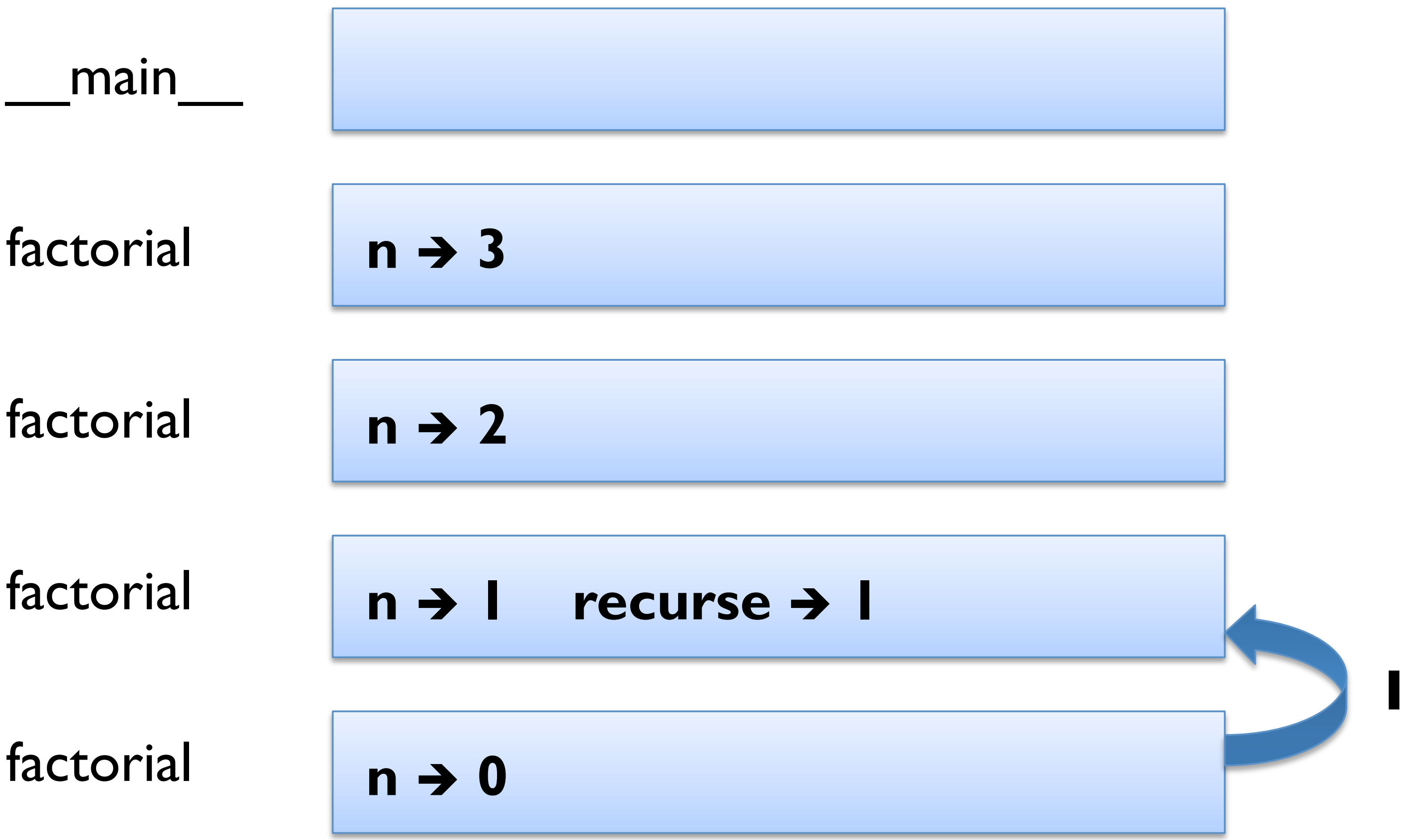

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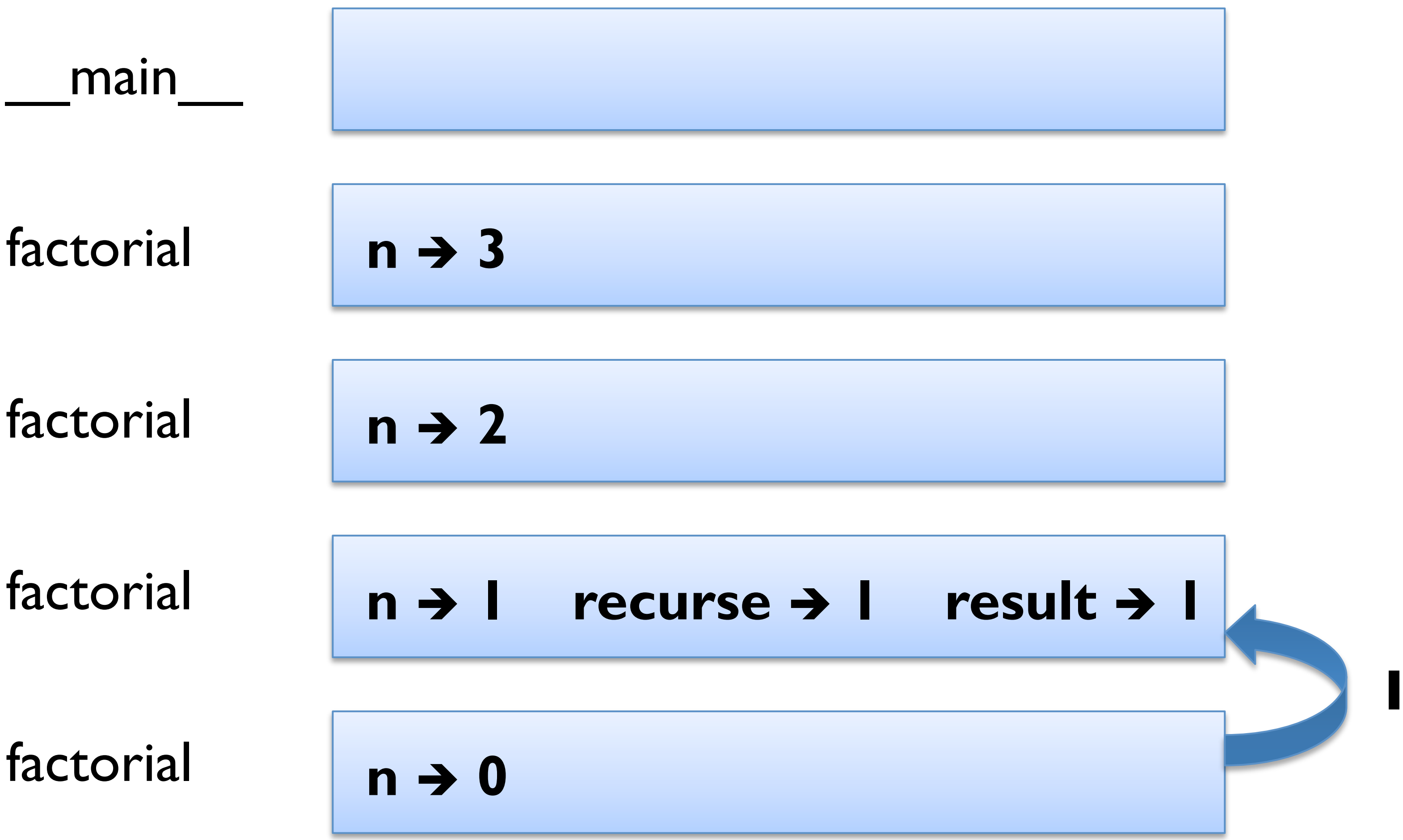

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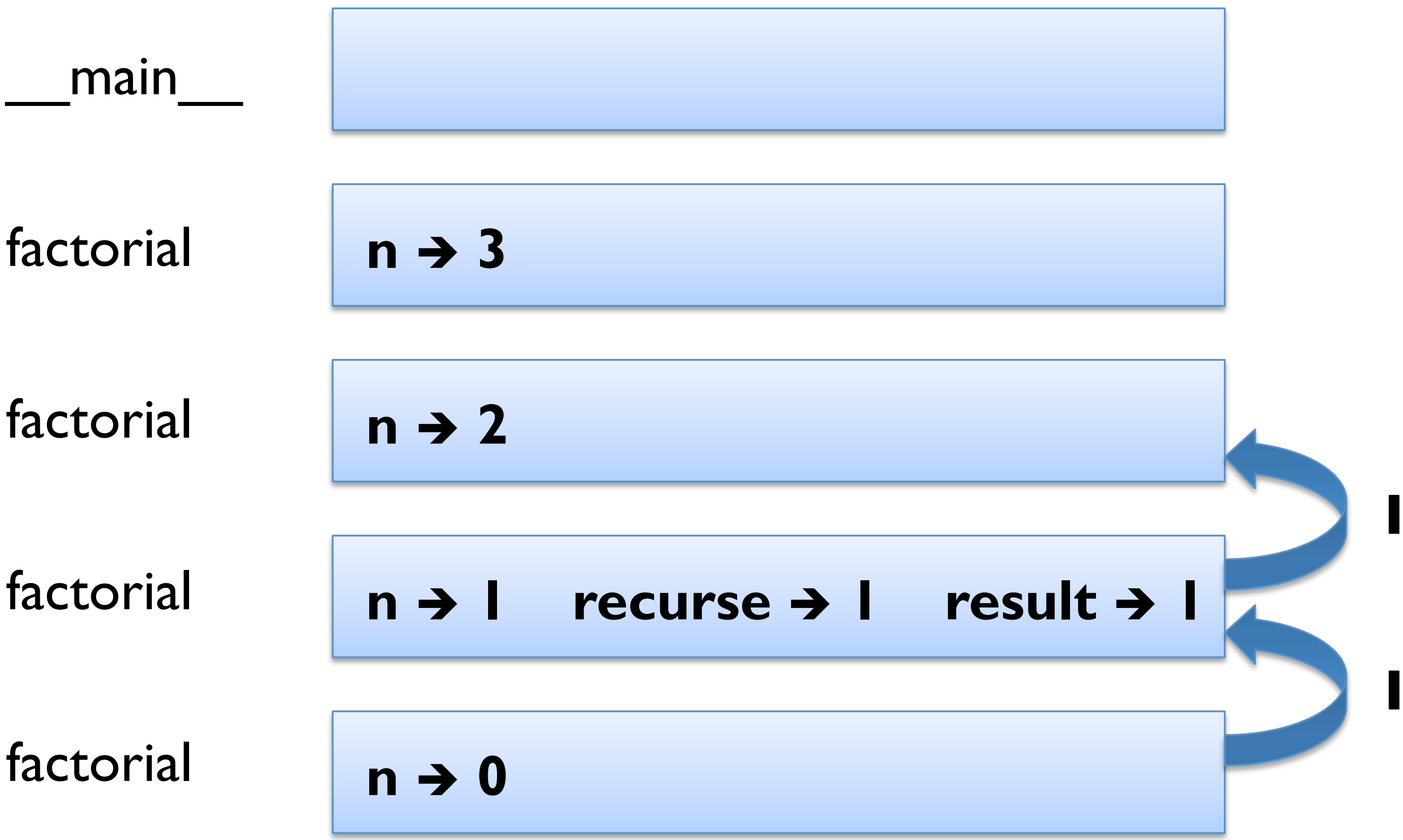

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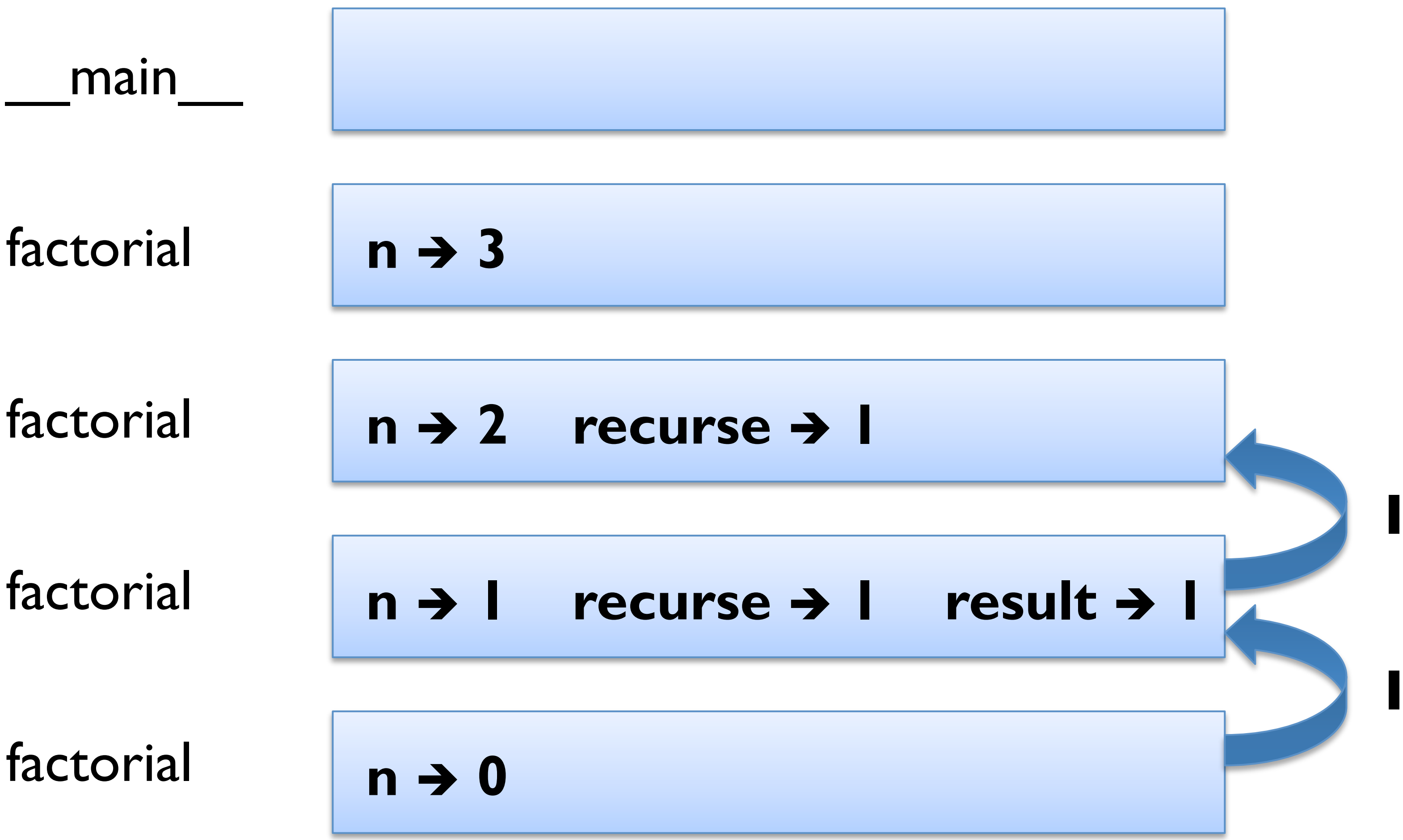

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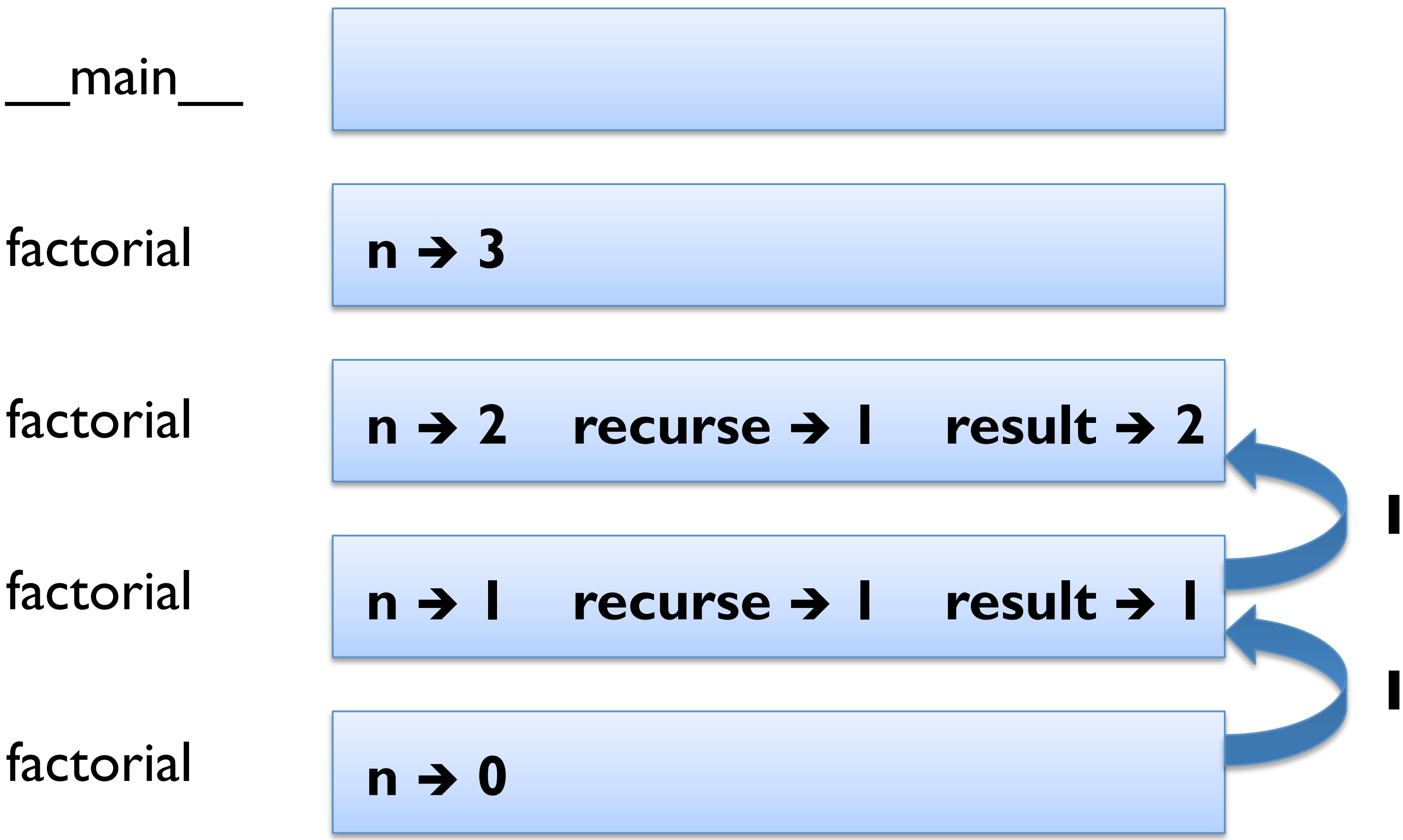

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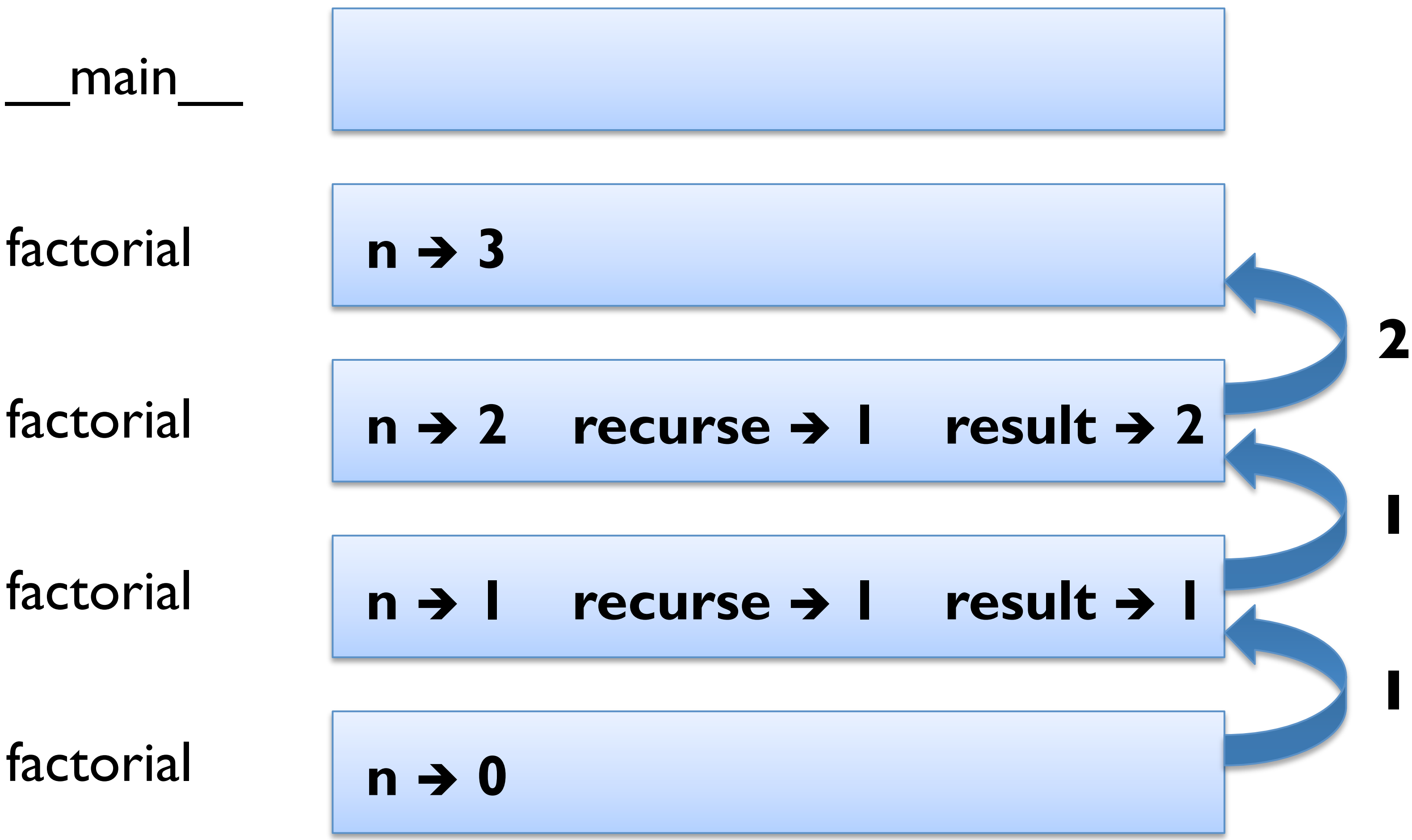

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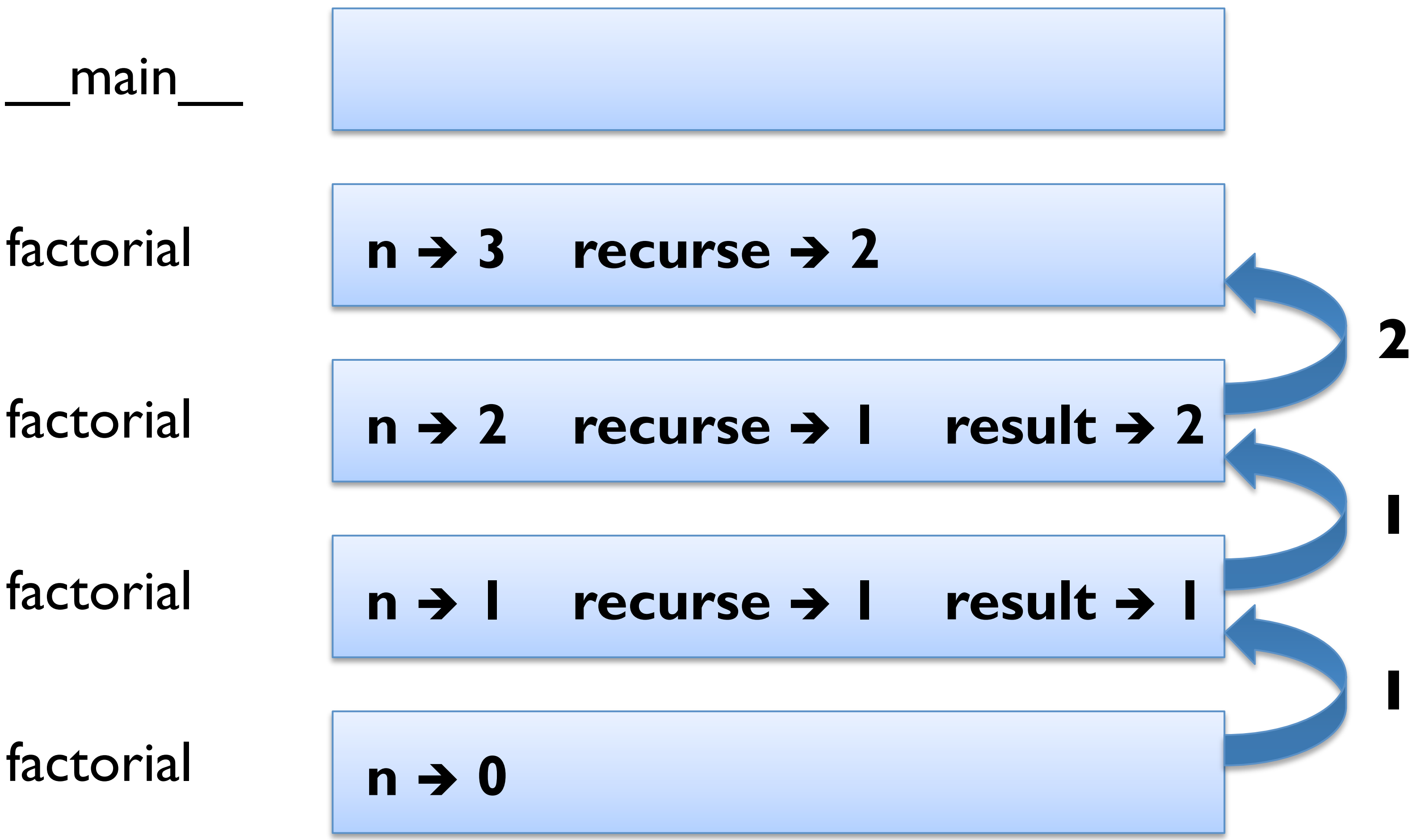

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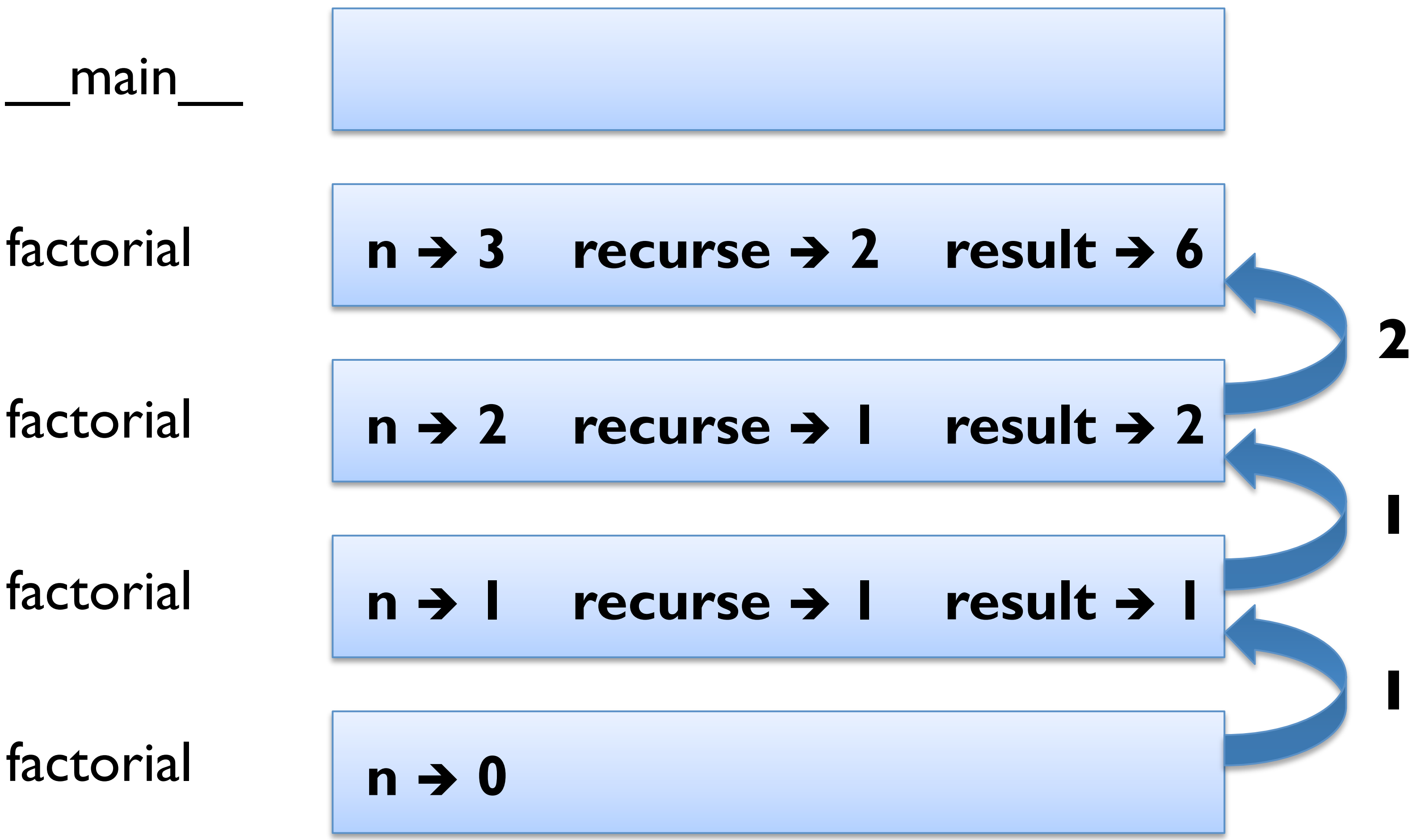

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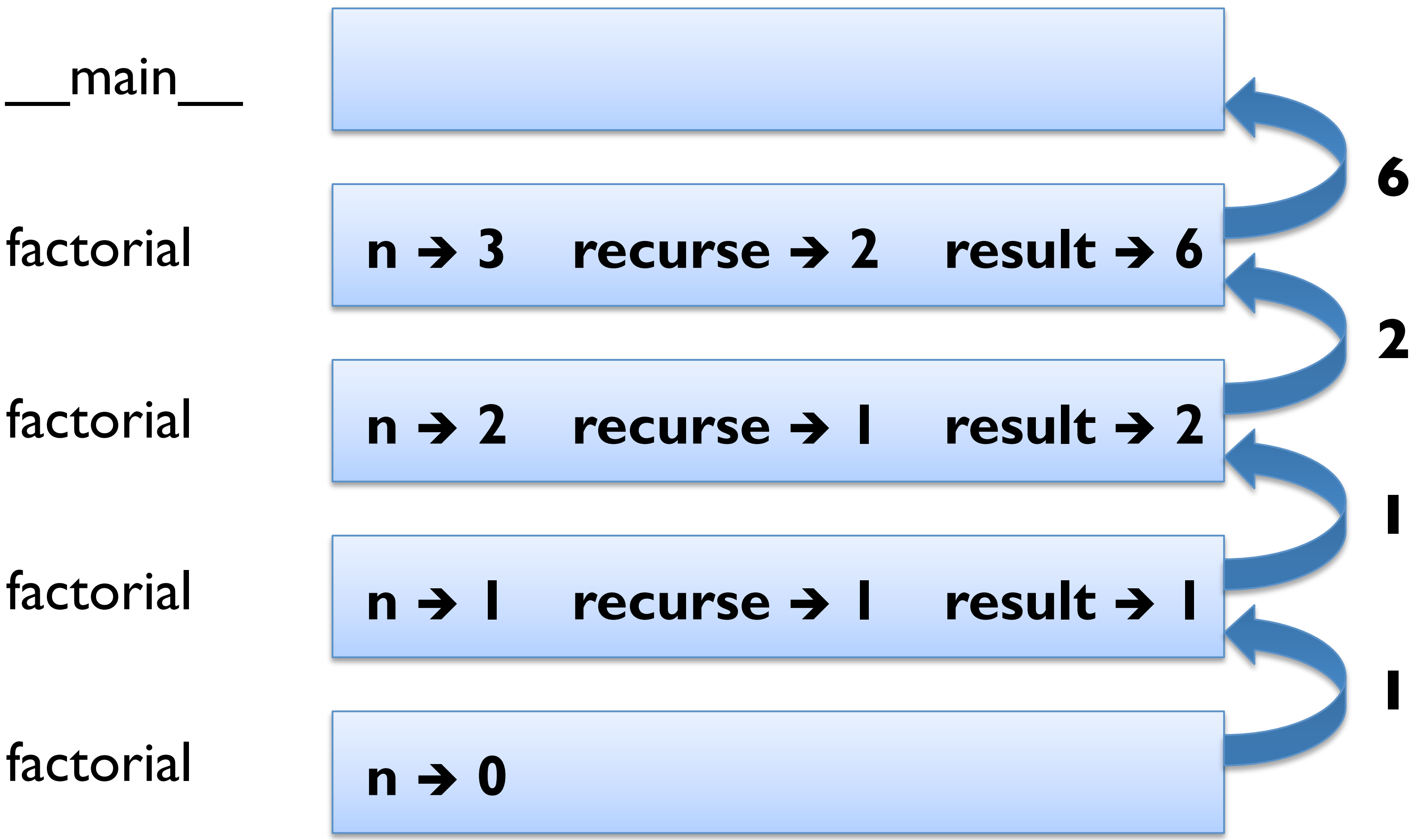

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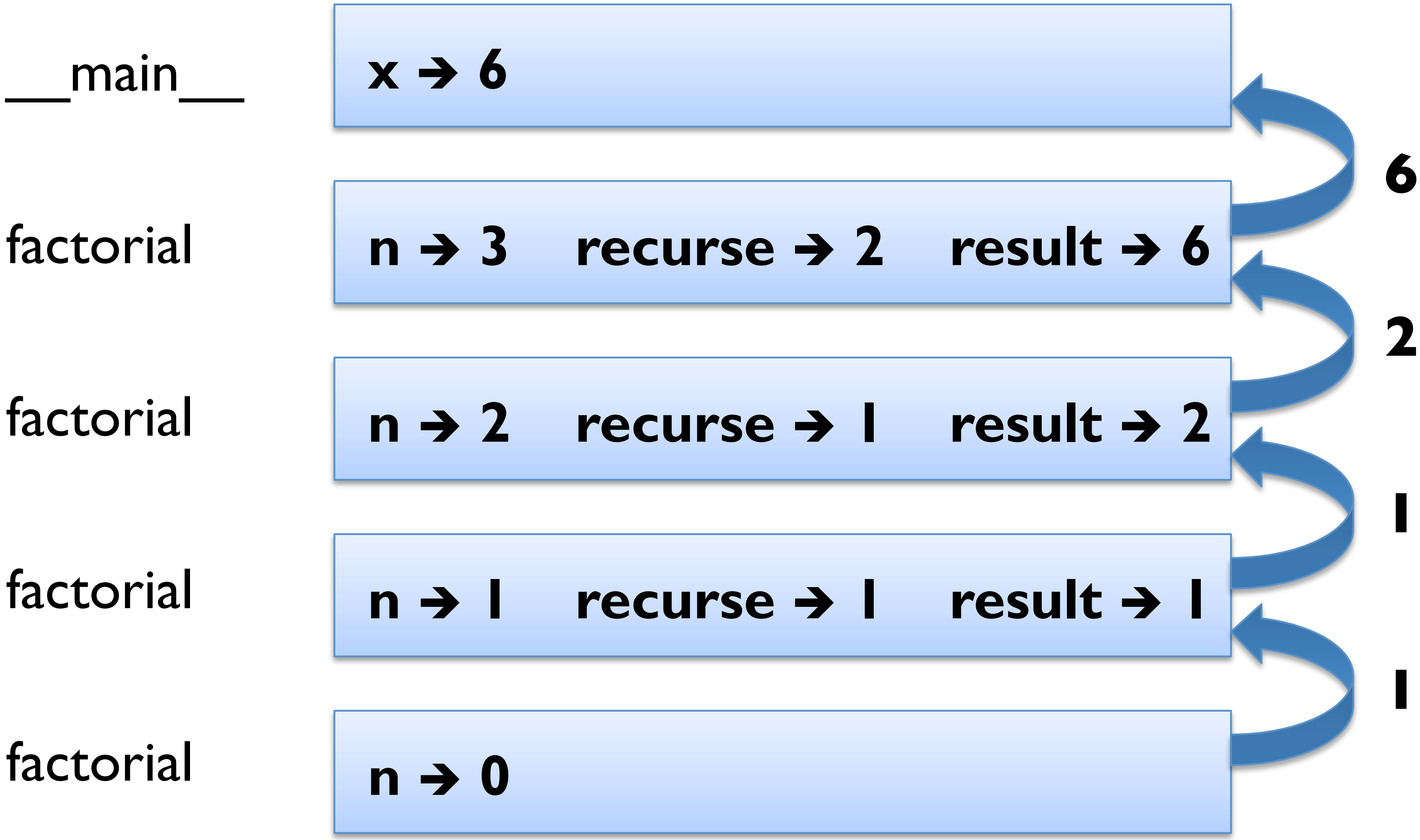


Leap of Faith

- following the flow of execution difficult with recursion
- alternatively take the "leap of faith" (*induction*)

- Example:

```
def factorial(n):
    if n == 0:
        return 1
    recurse = factorial(n-1)
    result = n * recurse
    return result
x = factorial(3)
```

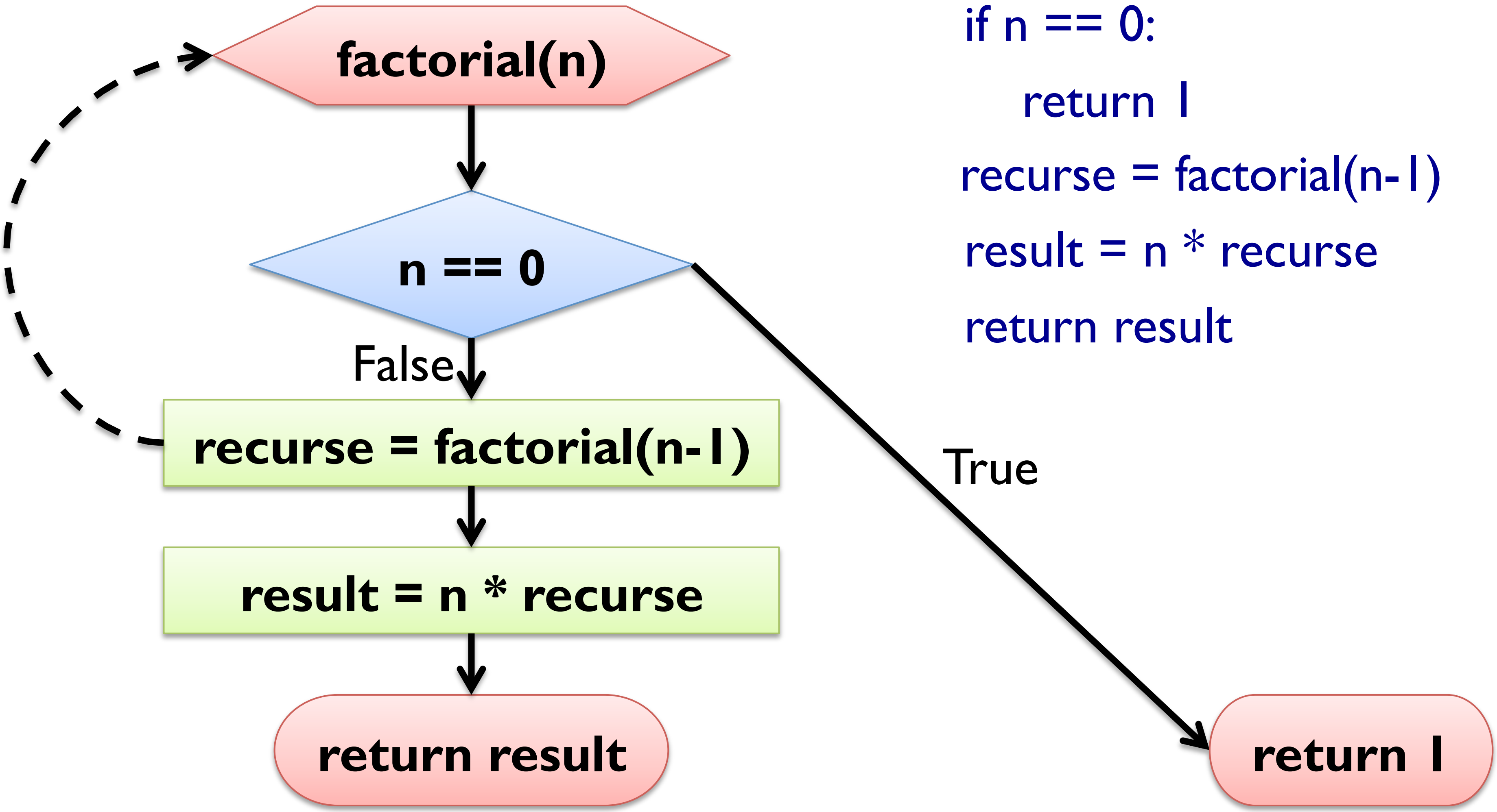
check the base case

assume recursive call is correct

check the step case

Control Flow Diagram

- Example:




```
def factorial(n):
    if n == 0:
        return 1
    recurse = factorial(n-1)
    result = n * recurse
    return result
```

Fibonacci

- Fibonacci numbers model for unchecked rabbit population
- rabbit pairs at generation n is sum of rabbit pairs at generation n-1 and generation n-2
- mathematically:
  - fib(0) = 0, fib(1) = 1, fib(n) = fib(n-1) + fib(n-2)
- Pythonically:

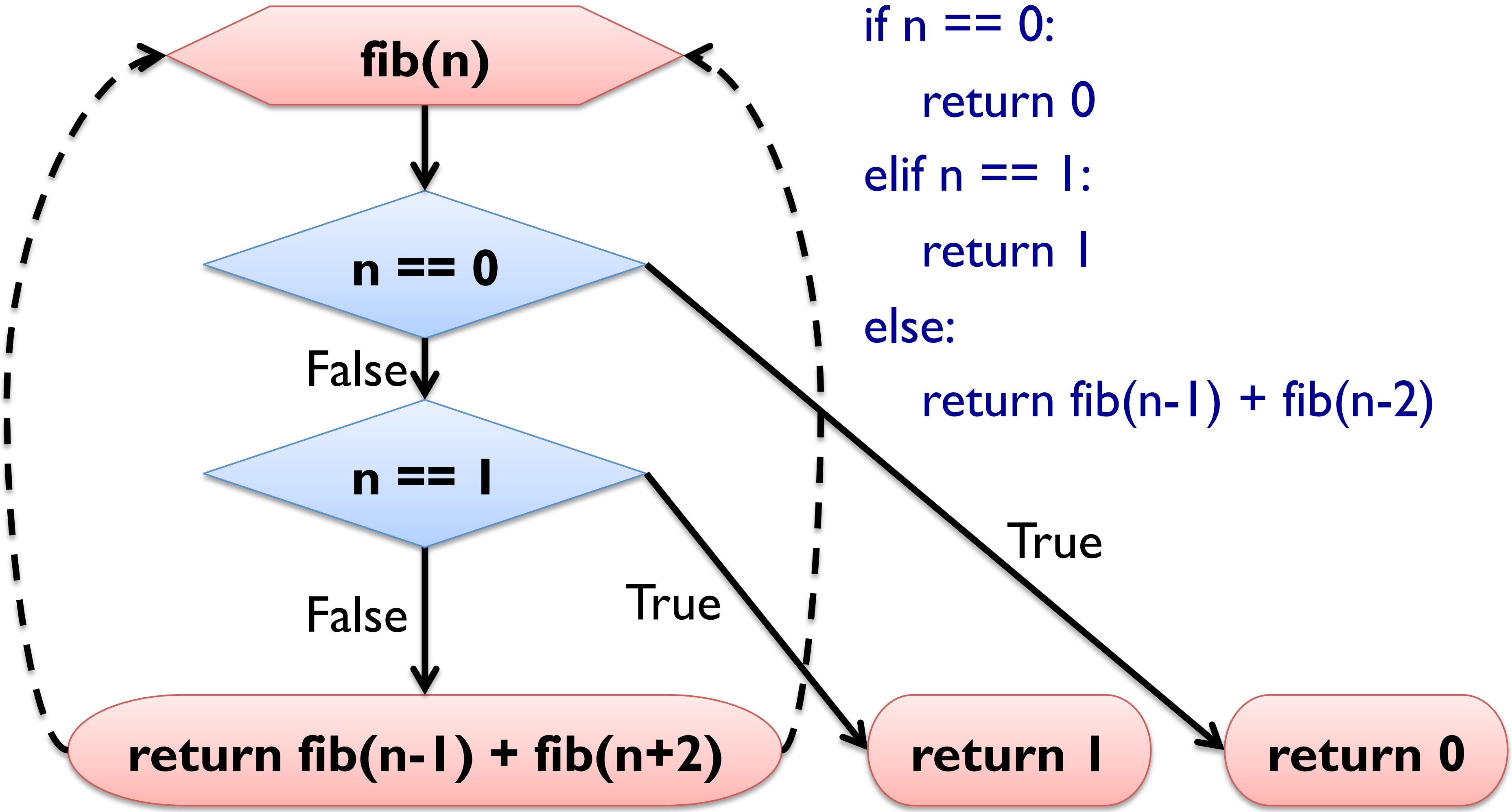
```
def fib(n):
    if n == 0:    return 0
    elif n == 1:  return 1
    else:         return fib(n-1) + fib(n-2)
```

- "leap of faith" required even for small n!

Control Flow Diagram

- Example:

```
def fib(n):
  if n == 0:
     return 0
  elif n == 1:
     return 1
  else:
     return fib(n-1) + fib(n-2)
```



Types and Base Cases

```
def factorial(n):
    if n == 0:
        return 1
    recurse = factorial(n-1)
    result = n * recurse
    return result
```

- **Problem:** factorial(1.5) exceeds recursion limit
- factorial(0.5)
- factorial(-0.5)
- factorial(-1.5)
- …

Types and Base Cases

```
def factorial(n):
    if n == 0:
        return 1
    recurse = factorial(n-1)
    result = n * recurse
    return result
```

- **Idea:** check type at beginning of function

Types and Base Cases

```
def factorial(n):
    if not isinstance(n, int):
        print "Integer required";  return None
    if n == 0:
        return 1
    recurse = factorial(n-1)
    result = n * recurse
    return result
```

- **Idea:** check type at beginning of function

Types and Base Cases

```
def factorial(n):
    if not isinstance(n, int):
        print "Integer required";  return None
    if n < 0:
        print "Non-negative number expected";  return None
    if n == 0:
        return 1
    recurse = factorial(n-1)
    result = n * recurse
    return result
```

- **Idea:** check type at beginning of function

Debugging Interfaces

- interfaces simplify testing and debugging

1. test if pre-conditions are given:
   - do the arguments have the right type?
   - are the values of the arguments ok?
2. test if the post-conditions are given:
   - does the return value have the right type?
   - is the return value computed correctly?
3. debug function, if pre- or post-conditions violated

Debugging (Recursive) Functions

- to check pre-conditions:
  - print values & types of parameters at beginning of function
  - insert check at beginning of function (*pre assertion*)
- to check post-conditions:
  - print values before return statements
  - insert check before return statements (*post assertion*)
- side-effect: visualize flow of execution

ITERATION

Multiple Assignment Revisited

- as seen before, variables can be assigned multiple times
- assignment is NOT the same as equality
- it is not symmetric, and changes with time

- Example:

```
a = 42
…
b = a
…
a = 23
```

from here, a and b are equal

from here, a and b are different

Updating Variables

- most common form of multiple assignment is *updating*
- a variable is assigned to an expression containing that variable

- Example:

```
x = 23
for i in range(19):
    x = x + 1
```

- adding one is called *incrementing*

- expression evaluated BEFORE assignment takes place
- thus, variable needs to have been *initialized* earlier!

Iterating with While Loops

- iteration = repetition of code blocks
- can be implemented using recursion (countdown, polyline)

- while statement:

  <while-loop> => while <cond>:

  <instr$_1$>; <instr$_2$>; <instr$_3$>

- Example:

```
def countdown(n):
    while n > 0:
        print n, "seconds left!"
        n = n - 1
    print "Ka-Boom!"
countdown(3)
```

n == 0

False

Termination

- Termination = the condition is eventually False
- loop in countdown obviously terminates:

```
while n > 0:        n = n - 1
```

- difficult for other loops:

```
def collatz(n):
    while n != 1:
        print n,
        if n % 2 == 0:          # n is even
            n = n / 2
        else:                   # n is odd
            n = 3 * n + 1
```

Termination

- Termination = the condition is eventually False
- loop in countdown obviously terminates:

```
while n > 0:    n = n - 1
```

- can also be difficult for recursion:

```
def collatz(n):
    if n != 1:
        print n,
        if n % 2 == 0:          # n is even
            collatz(n / 2)
        else:                   # n is odd
            collatz(3 * n + 1)
```

Breaking a Loop

- sometimes you want to *force* termination
- Example:

```
while True:
    num = raw_input('enter a number (or "exit"):\n')
    if num == "exit":
        break
    n = int(num)
    print "Square of", n, "is:", n**2
print "Thanks a lot!"
```

Approximating Square Roots

- Newton's method for finding root of a function f:
  1. start with some value x_0
  2. refine this value using $x_{n+1} = x_n - f(x_n) / f'(x_n)$
- for square root of a: $f(x) = x^2 - a$ $f'(x) = 2x$
- simplifying for this special case: $x_{n+1} = (x_n + a / x_n) / 2$

- Example 1:

```
while True:
    print xn
    xnp1 = (xn + a / xn) / 2
    if xnp1 == xn:
        break
    xn = xnp1
```

Approximating Square Roots

- Newton's method for finding root of a function f:
  1. start with some value x_0
  2. refine this value using $x_{n+1} = x_n - f(x_n) / f'(x_n)$

- Example 2:

```
def f(x):       return x**3 - math.cos(x)
def f1(x):      return 3*x**2 + math.sin(x)
while True:
    print xn
    xnp1 = xn - f(xn) / f1(xn)
    if xnp1 == xn:
        break
    xn = xnp1
```

Approximating Square Roots

- Newton's method for finding root of a function f:
  1. start with some value x_0
  2. refine this value using $x_{n+1} = x_n - f(x_n) / f'(x_n)$

- Example 2:

```
def f(x):      return x**3 - math.cos(x)
def f1(x):     return 3*x**2 + math.sin(x)
while True:
    print xn
    xnp1 = xn - f(xn) / f1(xn)
    if math.abs(xnp1 - xn) < epsilon:
        break
    xn = xnp1
```

Algorithms

- algorithm = mechanical problem-solving process
- usually given as a step-by-step procedure for computation

- Newton's method is an example of an algorithm
- other examples:
  - addition with carrying
  - subtraction with borrowing
  - long multiplication
  - long division

- directly using Pythagora's formula is not an algorithm

Divide et Impera

- latin, means "divide and conquer" (courtesy of Julius Caesar)
- **Idea:** break down a problem and recursively work on parts
- Example: guessing a number by bisection

```
def guess(low, high):
    if low == high:
        print "Got you! You thought of: ", low
    else:
        mid = (low+high) / 2
        ans = raw_input("Is "+str(mid)+" correct (>, =, <)?")
        if ans == ">":      guess(mid,high)
        elif ans == "<":    guess(low,mid)
        else:               print "Yeehah! Got you!"
```

Debugging Larger Programs

- assume you have large function computing wrong return value
- going step-by-step very time consuming

- **Idea:** use bisection, i.e., half the search space in each step

1. insert intermediate output (e.g. using print) at mid-point
2. if intermediate output is correct, apply recursively to 2nd part
3. if intermediate output is wrong, apply recursively to 1st part

STRINGS

Strings as Sequences

- strings can be viewed as 0-indexed sequences
- Examples:

  "Slartibartfast"[0] == "S"

  "Slartibartfast"[1] == "l"

  "Slartibartfast"[2] == "Slartibartfast"[7]

  "Phartiphukborlz"[-1] == "z"

- grammar rule for expressions:

  <expr> => … | <expr$_1$>[<expr$_2$>]

- <expr$_1$> = expression with value of type string
- index <expr$_2$> = expression with value of type integer
- negative index counting from the back

Length of Strings

- length of a string computed by built-in function len(object)

- Example:

```
name = "Slartibartfast"
length = len(name)
print name[length-4]
```

- Note: name[length] gives runtime error

- identical to write name[len(name)-1] and name[-1]
- more general, name[len(name)-a] identical to name[-a]