DM573 Introduction to Computer Science Exercises on Satisfiability

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PART I

Exercise I-1: Check Satisfiability

Which of the following formulas are satisfiable (give a satisfying assignment)? Which are not (give reasons)?

Note: in all exercises, logical negation (NOT) is denoted by "-".

- a) A ^ B
- b) A v B
- c) A → B
- d) A ^ –A
- e) Av-A

Exercise I-2: Equivalent Formulas

Two formulas are equivalent, if the same assignments satisfy both of them.

Which of the following formulas are equivalent?

- a) –A ^ B
- b) –A v B
- c) A → B
- d) $(A \rightarrow B) \land (-B \rightarrow A)$
- e) $(-A \rightarrow B) \land (-B \rightarrow -A)$

Exercise I-3: Convert to CNF

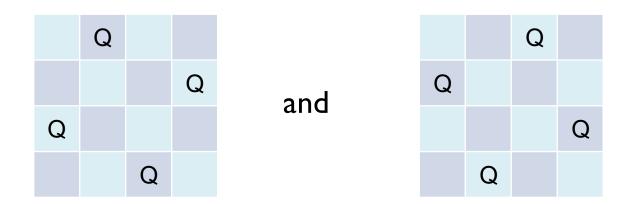
Convert the following formulas into CNF:

- a) –A∧B
- b) –A v B
- c) A → B
- d) $(A \rightarrow B) \land (-B \rightarrow A)$

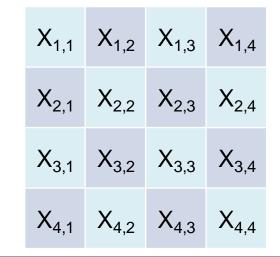
e) A → (- (B ∨ D))

Exercise I-4: Breaking Symmetry

Solutions to N-Towers and N-Queens are symmetric:



Write two clauses that forbid solutions where there is a queen in the right half of the first row.



Exercise I-5: Preparation

- Install lingeling or another compatible SAT solver
- Alternatively, use a Javascript SAT solver, e.g.:
 - https://www.msoos.org/2013/09/minisat-in-your-browser/
- Test it using the following input (also available on the course web page as the file towers2x2.cnf)

```
p cnf 4 6
-1 -2 0
-1 -3 0
-2 -4 0
-3 -4 0
1 2 0
3 4 0
```

Exercise I-6: Removing Redundancies

The formula from page 11 on Peter's slides contains redundant information. For example, $X_{1,1} \rightarrow -X_{1,2}$ and $X_{1,2} \rightarrow -X_{1,1}$ are equivalent. Understand and remove these redundancies:

- a) Why do these redundancies occur?
- b) Identify all such redundancies!
- c) Write down a simplified formula without redundancies!
- d) Convert the simplified formula into CNF!
- e) Write the formula in DIMACS format!
- f) Run the lingeling or another SAT solver on it and interpret the result!

PART II

Exercise II-1: Check Satisfiability

Which of the following formulas are satisfiable (give a satisfying assignment)? Which are not (give reasons)?

- a) $(A \rightarrow B) \land (B \rightarrow A)$
- b) $(A \rightarrow B) \land (B \rightarrow A) \land A$
- c) $(A \rightarrow B) \land (B \rightarrow A) \land -A$
- d) $(A \rightarrow B) \land (B \rightarrow -A) \land (-A \rightarrow -B) \land (-B \rightarrow A)$

Exercise II-2: Equivalent Formulas

Two formulas are equivalent, if the same assignments satisfy both of them.

Which of the following formulas are equivalent?

- a) $(A \rightarrow B) \land (-B \rightarrow A)$
- b) $(A \rightarrow -B) \land (B \rightarrow A)$
- c) (−A ∨ −B) ∧ (A ∨ −B)
- d) $(B \lor A) \land (-A \lor B)$

Exercise II-3: Convert to CNF

Convert the following formulas into CNF:

- a) $(-A \rightarrow B) \land (-B \rightarrow -A)$
- b) A → (– (B ∧ D)
- c) $A \rightarrow (-(B \rightarrow (C \land D)))$

Exercise II-4: 3-Towers

Write a Python program that generates the input for a SAT solver to solve the 3-Towers problem:

- a) Write a function pair2int(r,c) which maps (1,1), (1,2), ..., (3,3) to 1 to 9 using the formula 3*(r-1)+C.
- b) Write nested for-loops that go through all positions on the board from (1,1) to (3,3) and produces clauses that represent attacks.
- c) Write a for-loop that produces clauses that specify that all 3 rows contain a tower.
- d) Using (a)–(c), write a DIMACS file and test it using lingeling or another SAT solver.

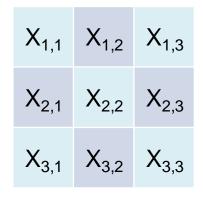
X _{1,1}	X _{1,2}	X _{1,3}
X _{2,1}	X _{2,2}	X _{2,3}
X _{3,1}	X _{3,2}	X _{3,3}

1	2	3
4	5	6
7	8	9

Exercise II-5: N-Towers

Generalize your Python program from Exercise II-4 to generate the input for a SAT solver to solve the N-Towers problem:

- a) Write a function pair2int (n, r, c)
 which maps pairs (r,C) to the integers 1 to N²
 using the formula N*(r-1)+C.
- b) Write nested for-loops that go through all positions on the board from (1,1) to (n,n) and produces clauses that represent attacks.
- c) Write a for-loop that produces clauses that specify that all rows contain a tower.
- d) Using (a)–(c), write a DIMACS file and test it using lingeling or another SAT solver.

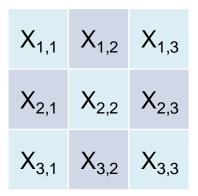


1	2	3
4	5	6
7	8	9

Exercise II-6: N-Queens

Extend your Python program from Exercise II-5 to generate the input for a SAT solver to solve the N-Queens problem:

- a) Reuse your function pair2int(n,r,c) from Exercise II-5.
- b) Adapt your for-loops from Exercise II-5 to produce also clauses for the diagonals.
- c) Reuse the for-loop from Exercise II-5 that produces clauses that specify that all rows contain a tower.
- d) Using (a)–(c), write a DIMACS file and test it using lingeling or another SAT solver.



1	2	3
4	5	6
7	8	9