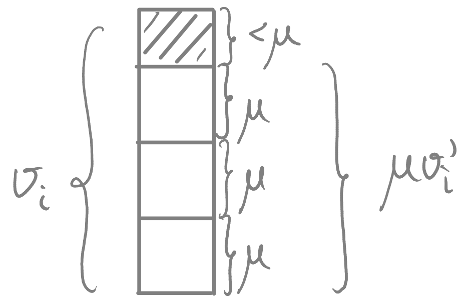
Proof:

Approximation ratio:

For each item i, $\mu v_i'$ equals v_i rounded down to the nearest multiple of μ. (*)

Thus, $v_i - \mu v_i' < \mu$ (each item „loses" less than μ in the rounding.) (**)



Let S be the set of items selected by Alg. 3.2

This is an optimal solution to the instance with values v_i', and hence, to the instance with values $\mu v_i'$. (***)

Let O be the set of items in an optimal solution to the original instance with values v_i.

The total value produced by Alg. 3.2 is

$$
\begin{aligned}
\sum_{i \in S} v_i &\geq \sum_{i \in S} \mu v_i', && \text{by } (*) \\
&\geq \sum_{i \in O} \mu v_i', && \text{by } (***) \\
&> \sum_{i \in O} (v_i - \mu), && \text{by } (**) \\
&\geq \left(\sum_{i \in O} v_i\right) - n\mu, && \text{since } |O| \leq n \\
&= OPT - \varepsilon M \\
&\geq (1-\varepsilon)\, OPT, && \text{since } OPT \geq M
\end{aligned}
$$

Running time:
See above.

$\square$

According to Thm 3.5, Alg. 3.2 is a

fully polynomial time approximation scheme (FPTAS)

also poly. in $1/\varepsilon$

poly. in input size

Family $\{A_\varepsilon\}$ of alg., where A_ε has precision ε.

($(1-\varepsilon)$-approx. alg for max. problems,
$(1+\varepsilon)$-approx. alg for min. problems)

Def. 3.4

Def. 3.3

Thus, Thm 3.5 could also be stated like this:

$\boxed{\text{Theorem 3.5: Alg 3.2 is a FPTAS}}$

In the Multiple Knapsack problem, there are a fixed number of knapsacks.

Bin Packing can be seen as a dual problem of Multiple Knapsack:

In the $\boxed{\text{Bin Packing}}$ problem, there is an unlimited supply of bins, all of size 1. The aim is to pack all items in as few bins as possible.

Simple approx. alg.s:	Asymptotic approx. ratio
Next-fit (NF)	2
First-Fit (FF)	1.7
Best-Fit (BF)	1.7
Next-Fit-Decreasing (NFD)	≈ 1.69
First-Fit-Decreasing (FFD)	1.222...
Best-Fit-Decreasing (BFD)	1.222...

Approx. scheme?

Can we do the same kind of rounding for Bin Packing as we did for Knapsack?

Section 3.3 : The Bin Packing Problem

Last time we discussed simple approx. alg.s
Today we will develop an approximation scheme.

Approximation scheme $\{A_\varepsilon\}$:

1. Transform $I \to I''$:
   a. Remove all items smaller than $\varepsilon/2$. $(I \to I')$
      $\Rightarrow O(\frac{1}{\varepsilon})$ items fit in one bin
   b. Round up sizes of remaining items $(I' \to I'')$
      $\Rightarrow O(1)$ different item sizes
2. Do dyn. prg. on I''
   $\Rightarrow A_\varepsilon(I'') = OPT(I'')$
3. Add small items to the packing using First-Fit (or any other Anyfit alg.)

Adding small items to the packing (3.)

Lemma 3.10

$$A_\varepsilon(I) \le \max\left\{A_\varepsilon(I''),\ \frac{2}{2-\varepsilon}\,\mathrm{size}(I)+1\right\}$$

Proof:

If no extra bin is needed for adding the small items, $A_\varepsilon(I) = A_\varepsilon(I'')$.

Otherwise, all bins, except possibly the last one, are filled to more than $1-\varepsilon/2$.

In this case,

$$A_\varepsilon(I) \le \left\lceil \frac{\mathrm{size}(I)}{1-\varepsilon/2} \right\rceil \le \frac{\mathrm{size}(I)}{1-\varepsilon/2} + 1$$

$$= \frac{2}{2-\varepsilon}\,\mathrm{size}(I) + 1$$

$\square$